CSU Math 301
Fall 2018

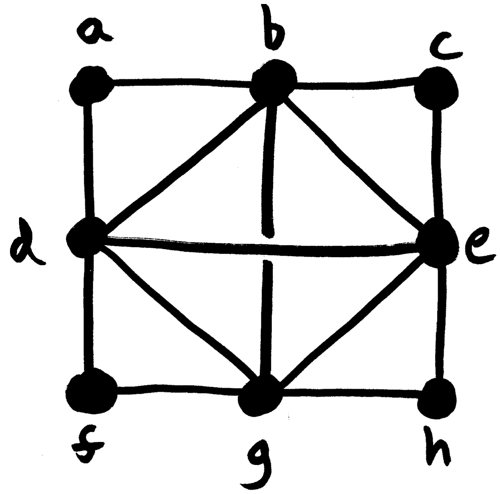
Practice Homework 13

Due never!

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Show that the graph drawn below is not 3-colorable but is 4-colorable.



2. Let G be a connected graph such that all vertices except $d+1$ have degree at most d (the remaining $d+1$ vertices may have degree larger than d). Prove that G is $(d+1)$-colorable.

3. Show that a graph G is a tree if and only if it contains no cycles, but adding any new edge creates a cycle. This is part (b) of Theorem 8.1.1.

4. Let G be a connected weighted graph with positive edge costs.

   (a) Describe how to find a spanning tree for which the sum of the edge-costs is *maximal.*

   Hint: Create a new weighted graph G' by multiplying each edge weight of G by -1.

   (b) Describe how to find a spanning tree for which the *product* of the edge-costs is minimal.

   Hint: Create a new weighted graph G' by editing the edge weights of G using logarithms.

5. Show by an example that if we don't assume the triangle inequality, then a tour found by the Tree Shortcut Algorithm can be longer than 1000 times an optimal tour.