Name: $\qquad$

- Unless stated otherwise, explain your logic and write out complete sentences.
\#4 is a short answer question. If you write the correct mathematical expression then you will get $100 \%$ credit even without any English words (but clear explanations may help you get more partial credit if you don't).
For \#5, just say "True" or "False". No partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
"I will not give, receive, or use any unauthorized assistance."

Signature:

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| Total | 50 |  |

## Practice Midterm 2C

1 (a) ( 7 points) Find an integer $x$ between 0 and 78 such that $12 x+2 \equiv 5 \bmod 79$.

## Practice Midterm 2C

(b) (3 points) Let $a$ and $b$ be integers (perhaps negative) and let $m$ be a positive integer. Give a precise definition of what it means to have $a \equiv b \bmod m$.

## CSU Math 301

## Practice Midterm 2C

2 You have $\$ 9$ dollars to spend. Each day at lunch you buy exactly one item: either an apple for $\$ 1$, or a yogurt for $\$ 2$, or a sandwich for $\$ 4$. You continue buying one item each day until you have exactly $\$ 0$. This could take anywhere from 9 days, if you buy 9 apples in a row, to 3 days, if you buy 2 sandwiches in a row then an apple. Buying two sandwiches then an apple is different from buying an apple then two sandwiches. In how many different ways could you spend your money?

## CSU Math 301

## Practice Midterm 2C

3 (8 points) Prove that there are an infinite number of primes.
(2 points) True or False (no justification needed, and no partial credit): If we label the prime numbers in order as $p_{1}=2, p_{2}=3, p_{3}=5, p_{4}=7, p_{5}=11, \ldots$, etc, then for any positive integer $k$ there exists some $i$ such that $p_{i}-p_{i-1} \geq k$.

## CSU Math 301

## Practice Midterm 2C

4 Short answer questions. No English words required (except perhaps for partial credit).
(a) (3 points) How many 5-digit strings of digits 0-9 are there with no two consecutive digits the same? For example, strings 01323 and 93572 are allowed, but 00232 (consecutive 0's) and 93552 (consecutive 5's) are not.
(b) (3 points) How many ways are there to place 8 rooks on a chessboard with no two attacking each other if all 8 rooks are made of different materials? That is, the rooks are all distinguishable from each other.

## Practice Midterm 2C

(c) (4 points) How many graphs are there with exactly 7 vertices $a, b, c, d, e, f, g$ and 3 edges? The graph with edges $a b, a c, b d$ is considered different from the graph with edges $a b, a c, b e$.

## CSU Math 301

## Practice Midterm 2C

5 No justification needed: just say "True" or "False". No partial credit.
(a) True or False: Let $a$ and $b$ be positive integers. If $\operatorname{gcd}(a, b) \mid c$, then there exist integers $m$ and $n$ with $c=m a+n b$.
(b) True or False: Let $G$ and $H$ be graphs. If $G$ is the complement of $H$, then $H$ is the complement of $G$.
(c) True or False: If $a \mid c$ and $b \mid c$ and $a>b$, then $(a-b) \mid c$.
(d) True or False: There exists a connected graph $G$ with vertices of degrees $2,2,2,3,3,4,4,4,6$ that contains an Eulerian walk starting at a vertex of degree 2 and ending at a vertex of degree 4.
(e) True or False: The number of 3-of-a-kind poker hands is $13 \cdot\binom{4}{3} \cdot 12 \cdot\binom{4}{1} \cdot 11 \cdot\binom{4}{1}$.
(Recall one example 3-of-a-kind poker hand is $\{K \diamond, K \diamond, K \boldsymbol{\uparrow}, 5 \boldsymbol{\downarrow}, 2 \diamond\}$, and neither a full house nor a 4-of-a-kind is considered to be 3-of-a-kind hand.)

CSU Math 301

## Practice Midterm 2C

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