Name: _____

• Unless stated otherwise, explain your logic and write out complete sentences.

#1(b) and #4 are short answer questions. If you write the correct mathematical expression then you will get 100% credit even without any English words (but clear explanations may help you get more partial credit if you don't).

For #5, just say "True" or "False". No partial credit is available.

- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:

"I will not give, receive, or use any unauthorized assistance."

Signature:

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Practice Midterm 2B

1 (a) (8 points) Let F_n be the *n*-th Fibonacci number. Prove that $F_1^2 + \ldots + F_n^2 = F_n F_{n+1}$ for all $n \ge 1$.

(b) (2 points) Short answer (no justification needed). What number do the successive quotients $\frac{F_2}{F_1}$, $\frac{F_3}{F_2}$, $\frac{F_4}{F_3}$, \ldots , $\frac{F_n}{F_{n-1}}$, $\frac{F_{n+1}}{F_n}$, \ldots approach as n gets larger and larger? Write the exact (simplified) expression for this number.

2

Practice Midterm 2B

(a) (6 points) Use the Euclidean algorithm to find an integer x between 0 and 42 that satisfies

 $8x \equiv 4 \mod 43.$

Practice Midterm 2B

(b) (4 points) How many graphs are there that have *exactly* 8 vertices $\{a, b, c, d, e, f, g, h\}$? We consider the graph with 8 vertices and a single edge ab to be different from the graph with 8 vertices and a single edge bc.

Practice Midterm 2B

3 (a) (5 points) Show that if integer n is not a perfect cube (meaning there is no integer m with $m^3 = n$), then in the prime factorization for n some prime appears with an exponent not divisible by 3.

(b) (5 points) Show that if integer n is not a perfect cube, then $\sqrt[3]{n}$ is irrational.

CSU Math 301 Practice Midterm 2B

- 4 Short answer questions. No English words required (except perhaps for partial credit).
 - (a) (3 points) You must assign the 12 months (Jan-Dec) to four nurses Alice, Bob, Carl, Diane so that each worker gets three months of "on call" duty. How many possible ways are there to do this?

(b) (4 points) How many subsets does the set $\{1, 2, 3, ..., 9\}$ have that contain no two consecutive integers?

(c) (3 points) How many one-pair poker hands are there? A one-pair poker hand, such as $\{6\heartsuit, 6\diamondsuit, 2\spadesuit, K\clubsuit, 9\diamondsuit\}$, consists of a pair of two cards of the same value, and then three cards of completely distinct values.

CSU Math 301 Practice Midterm 2B

- 5 No justification needed: just say "True" or "False". No partial credit.
 - (a) True or False: If $a \nmid b$ and $b \mid c$ then $a \nmid c$.

(b) True or False: If $a \nmid b$ and $b \nmid c$ then $a \nmid c$.

(c) True or False: The *complement* of the path graph on 5 vertices has an Eulerian walk.

(d) True or False: There exists a graph with 7 vertices such the total sum of all vertex degrees is 44.

(e) True or False: For every positive integer m and integer $1 \le a < m$, there exists some integer x such that $ax \equiv 1 \mod m$.

CSU Math 301 Practice Midterm 2B

This page intentionally left blank.