Name: _____

• For #1, #2, and #3, explain your logic fully and write complete sentences.

For #4, if you write the correct mathematical expression then you will get 100% credit even without any English words (but clear explanations may help you get more partial credit).

For #5, just say "True" or "False". No partial credit is available.

- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:

"I will not give, receive, or use any unauthorized assistance."

Signature:

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Show that there exist 1000 consecutive positive integers which are all composite (not prime).

(You are not allowed to say "This follows from the theorem that for every integer k there exist k consecutive composite numbers". You must reprove the special case when k = 1000.)

- 2 A staircase has n stairs, and you can take either one or three stairs at a time. Let S_n be the number of different ways you could climb to the top of the n stairs.
 - (a) (6 points) Find and fully justify a recurrence relation for S_n .

(b) (4 points) Find S_{10} , the number of ways to climb to the top of a staircase of 10 stairs.

CSU Math 301

Practice Midterm 2A

3 Use the Euclidean algorithm to find an integer x between 0 and 30 that satisfies

 $7x \equiv 10 \mod 31.$

- 4 No English words required (except perhaps for partial credit).
 - (a) (4 points) How many two-pair poker hands are there? A two-pair poker hand, such as $\{3\heartsuit, 3\diamondsuit, 5\clubsuit, 5\clubsuit, K\diamondsuit\}$, consists of a pair, a pair of a different value, and a 5th card of a different value.

(b) (3 points) Write down the exact integer that is equal to $\binom{10}{0} + \binom{10}{2} + \binom{10}{4} + \ldots + \binom{10}{8} + \binom{10}{10}$. There is a shortcut; I do not advise that you compute each term. You may use the fact that $2^{10} = 1024$.

(c) (3 points) You want to make a bag of 20 candies, and there are 4 different brands to choose from (Milky Way, Twix, Reese's, and Skittles). All candies of the same brand are the same. How many different bags could you create?

- 5 No justification needed: just say "True" or "False". No partial credit.
 - (a) True or False: There exists a graph with 7 vertices of degrees 2, 2, 2, 2, 2, 3, 7.

(b) True or False: A graph with n vertices always has at least 2^n subgraphs.

(c) True or False: Starting with 1, 1, 2, the Fibonacci numbers rotate *odd*, *odd*, *even*, then *odd*, *odd*, *even*, etc.

(d) True or False: The complete graph K_5 on 5 vertices contains a closed Eulerian cycle.

(e) True or False: For every positive integer a and positive integer b there exist integers m and n such that 1 = ma + nb.

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