

Name: _____

- For #1 and #2, write proofs that consist of complete sentences. For #3, #4, and #5, if you write the correct mathematical expression then you will get 100% credit even without any English words. But clear English explanations may help you get partial credit if you don't have the correct mathematical expression (and they may help you find the correct mathematical expression).
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
 "I will not give, receive, or use any unauthorized assistance."

Signature: _____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- 1 Suppose that 64 chairs are arranged in a *circle*. Prove that if 57 people sit in this circle of 64 chairs, then there are at least 9 consecutively filled chairs.

2 Prove that $2^0 + 2^1 + \dots + 2^n = 2^{n+1} - 1$ for all integers $n \geq 0$.

3 No English words required (except for partial credit).

- (a) You have 14 socks, all of different colors. You want to loan a subset of socks to your friend. The only requirement is that you must loan your friend an even number of socks. In how many different ways could you loan your friend an even number of socks?

(You may loan your friend 0 socks, 2 socks, \dots , or all 14 socks. If you loan your friend the red, blue, green, and yellow socks, then that is the same as loaning your friend the blue, red, green, and yellow socks, but of course different than loaning them the purple, black, grey, and brown socks.)

- (b) You have 14 socks, all of different colors. In how many different ways could you group the 14 socks into 7 pairs of two socks?

(The pair red–blue is the same as the pair blue–red. The grouping red–blue, green–yellow, \dots is the same as the grouping green–yellow, red–blue, \dots .)

4 No English words required (except for partial credit).

- (a) Suppose A and B are sets such that $|A \setminus B| = 17$, $|A \cap B| = 5$, and $|A \cup B| = 40$. What is $|B|$?

- (b) You want to make a bag of 15 lollipops, and there are 5 different flavors to choose from. How many different bags could you create?

(The bag with 10 grape, 4 orange, and 1 strawberry lollipop is the same as the bag with 4 orange, 10 grape, and 1 strawberry lollipop. All lollipops of the same flavor are the same.)

5 No English words required (except for partial credit).

(a) Suppose $\binom{n}{k} = 462$ and $\binom{n+1}{k} = 792$. What is $\binom{n}{k-1}$?

(b) Consider all anagrams of the word SPEEDED. You want to make a ranked list of your top 5 favorite anagrams, numbered #1 to #5. How many different ranked lists of 5 anagrams of SPEEDED are possible?