

Name: _____

- For #1 and #2, write proofs that consist of complete sentences. For #3, #4, and #5, if you write the correct mathematical expression then you will get 100% credit even without any English words. But clear English explanations may help you get partial credit if you don't have the correct mathematical expression.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
"I will not give, receive, or use any unauthorized assistance."

Signature: _____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- 1 Suppose you are given a set of 42 positive integers. Prove that at least 5 of these integers have the same 1's digit.

(Recall that the 1's digit of a positive integer is the last digit; for example the 1's digit of 834 is 4).

2 Prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for all integers $n \geq 1$.

3 No English words required (except for partial credit).

- (a) How many different ways are there to distribute 10 identical pieces of paper to Amy, Bob, and Carl? The only requirement is that all 10 pieces of paper need to be handed out.

- (b) Suppose Diane has 10 pencils (all distinct) and Eve has 6 erasers (all distinct). In how many different ways could Diane trade 4 of her pencils for 2 of Eve's erasers?

4 No English words required (except for partial credit).

- (a) In how many ways could you rank the top 10 restaurants (from #1 to #10) out of a collection of 50 restaurants?
- (b) You have 20 distinguishable books, and you want to donate a subset of them to the library. The only requirement is that you want to donate at least 1 book, and you don't want to donate all 20 books. How many different donations could you make?

5 No English words required (except for partial credit).

- (a) Suppose A and B are sets such that $|A| = 21$, $|B| = 15$, and $|A \cup B| = 30$. What is $|A \cap B|$?
- (b) Nine friends are sharing three hotel rooms, where the first hotel room fits 4 people, the second fits 3 people, and the third fits the remaining 2 people. They are staying in the hotel for 7 nights in a row, and they are allowed to rearrange rooms every night (repeating the same room assignments for different nights is allowed). In how many different ways can the 9 friends create a 7-night schedule of room assignments?