CSU Math 301 **Practice Final C**

Remark: The final exam will be comprehensive. Certainly not all topics from our class are represented on this practice final.

Name: ________________________________

- Unless stated otherwise, explain your logic and write out complete sentences.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

$\boxed{1}$ Short answer question. No explanations required. If you write the correct mathematical expression then you will get 100% credit even without any English words.

- (3 points) How many two-pair poker hands are there? A two-pair poker hand, such as $\{6\heartsuit, 6\diamondsuit, 2\spadesuit, 2\clubsuit, 9\diamondsuit\}$, consists of two pairs of two cards of the same value, and a fifth card of a different value. Four-of-a-kinds and full houses are not considered to be two-pair hands. The order of the cards doesn't matter.

- (3 points) In how many ways can you pair 26 people into 13 groups of 2? The group $\{A, B\}$ is the same as the group $\{B, A\}$, and the 13 groups are not ordered.

- (4 points) You want to create an ordered string with 10 symbols where exactly 7 symbols are numbers (0–9) and exactly 3 symbols are letters (a–z). String $abc1222233$ is different from $12c2a3b322$. How many such strings could you create?

$\boxed{2}$ Short answer question. No explanations required. If you write the correct mathematical expression then you will get 100% credit even without any English words.

How may ways are there to distribute 20 candies to 5 *distinguishable* people if . . .

- . . . the candies are identical? (2 points)

- . . . the candies are identical and every person must get at least one? (2 points)

- ...the candies are distinguishable and it is not required that every person get one? For example, we could give all 20 candies to the first person. (3 points)

- ...the candies are distinguishable and we give 8 to the first person, 4 to the second person, 3 to the third person, 3 to the fourth person, and 2 to the fifth person? (3 points)

$\boxed{3}$ Short answer question. No explanations required. If you write the correct mathematical expression then you will get 100% credit even without any English words.

- (4 points) If $\binom{n}{k} = 560$ and $\binom{n}{n-k+1} = 1820$, then what is $\binom{n+1}{k}$?

- (3 points) Let $n \geq 2$. How many labeled trees are there with exactly n vertices?

- (3 points) Assume $n \geq k$. How many ordered subsets of size k are there from a set of size n?

$\boxed{4}$ Suppose that 80 chairs are arranged in a *row*. Prove that if 75 people sit in this row of 80 chairs, then there are at least 13 consecutively filled chairs.

$\boxed{5}$ Let G be a connected weighted graph in which all edge weights are nonnegative. Prove that the cost of a minimal spanning tree is less than or equal to the cost of an optimal tour (i.e., a tour solving the Traveling Salesperson Problem).

$\boxed{6}$ Recall that the zeroth Fibonacci number is $F_0 = 0$, the first Fibonacci number is $F_1 = 1$, and the second Fibonacci number is $F_2 = 1$. Using (strong) induction, prove that for all integers $n \geq 1$ we have $F_n \leq 2^{n-1}$.

$\boxed{7}$ Show that a planar graph G with 10 vertices and 17 edges cannot be 2-colored.

$\boxed{8}$ Prove that $\sqrt{5}$ is not rational.

$\boxed{9}$ Prove that if a and b are positive integers with $b > a$, then $\gcd(a, b) = \gcd(a, b - a)$. I suggest explaining and verifying the following steps:

(i) Let $d = \gcd(a, b)$. Prove $d \mid a$ and $d \mid b - a$. Explain why $d \leq \gcd(a, b - a)$.

(ii) Let $d' = \gcd(a, b - a)$. By a similar process, show $d' \leq \gcd(a, b)$.

(iii) Conclude $\gcd(a, b) = \gcd(a, b - a)$.

$\boxed{10}$ No justification needed: just say "True" or "False". No partial credit.

(a) True or False: 757550 is a valid Prüfer code.

(b) True or False: If $a \equiv 0 \bmod b$ and $b \equiv 0 \bmod c$ then $a \equiv 0 \bmod c$.

(c) True or False: There exists a connected graph G with vertices of degrees 2, 2, 2, 2, 3, 4, 4, 6 that contains a non-closed Eulerian walk starting at the vertex of degree 3.

(d) True or False: A graph that is not d-colorable has a vertex of degree at least d.

(e) True or False: A simple graph (one in which each edge connects two distinct vertices, and in which there is at most one edge between any pair of vertices) with 30 edges has at least 9 vertices.

This page intentionally left blank.