#### Practice Final B

Remark: The final exam will be comprehensive. Certainly not all topics from our class are represented on this practice final.

Name: \_\_\_\_\_

- Unless stated otherwise, explain your logic and write out complete sentences.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement: "I will not give, receive, or use any unauthorized assistance."

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Signature:

#### Practice Final B

- 1 Short answer question. No explanations required. If you write the correct mathematical expression then you will get 100% credit even without any English words.
  - (a) You want to create a bouquet of 50 balloons, and there are 10 different colors to choose from. All balloons of the same color are the same, and there are an unlimited number of balloons of each color. How many different bouquets could you create?

(b) How many different strings of length 11 can you form by rearranging the letters of the word MISSISSIPPI? For example, SIMSISSIPPI and PIPISISIMSS are two such rearrangements.

(c) How many ways are there to place 8 rooks on a chessboard with no two attacking each other if 5 are wooden and 3 are marble? Rooks made from the same material are indistinguishable.

2 (a) Give the Prüfer code for the labeled tree drawn below.



(b) Draw the labeled tree for the Prüfer code 2426566.

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3 (a) In how many ways can you cover a  $3 \times 10$  grid of squares with identical dominoes, where each domino is of size  $3 \times 1$  (or  $1 \times 3$  if you turn it sideways), and you must use exactly 10 dominoes?

Remark: Each domino covers 3 squares, and so the 10 dominoes cover the 30 squares exactly.

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(b) Suppose 30 numbers are chosen from 1 to 82. Show that there is a group of at least 5 numbers such that the difference between any two numbers in this group is less than 12.

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4 (a) (8 points) Use to Euclidean algorithm to find an integer x between 0 and 92 such that  $11x \equiv 4 \mod 93$ .

(b) (2 points) True or False (no partial credit; no explanation needed): There is an integer x such that  $3x \equiv 1 \mod 9$ .

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5 Let  $F_n$  be the *n*-th Fibonacci number. Show that for all  $n \ge 1$  we have  $F_1 + F_3 + F_5 + \ldots + F_{2n-1} = F_{2n}$ .

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6 (a) Explain why the formula  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$  is true. In my opinion the easiest explanation is a combinatorial interpretation, but other explanations are possible.

(b) The binomial theorem gives a formula for  $(x+y)^n$  using the binomial coefficients. State this formula.

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(c) Let  $n \ge 1$ . What is  $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^{n-1}\binom{n}{n-1} + (-1)^n\binom{n}{n}$  equal to? This expression can be written in a much simpler way. Explain your answer.

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7 Prove that there are an infinite number of primes.

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8 Prove that a planar graph with v vertices (where  $v \ge 3$ ) has at most 3v - 6 edges.

Remark: I am not asking you to say "This is a theorem from the book"; I am asking you to prove this theorem.

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9 Let G be a connected graph such that all vertices except possibly d+1 have degree at most d (the remaining d+1 vertices may have arbitrarily large degree). Prove that G is (d+1)-colorable.

### CSU Math 301 Practice Final B

- 10 No justification needed: just say "True" or "False". No partial credit.
  - (a) True or False: If  $a \nmid b$  and  $a \nmid c$  then  $a \nmid (3b + 5c)$ .

(b) True or False: For any three sets A, B, and C we have  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

(c) True or False: If two edges in a connected weighted graph have the same edge-cost, then the graph has more than one minimal spanning tree.

(d) True or False: If  $a \equiv b \mod c$ , then for any integer x we have  $a + x \equiv b + x \mod (c + x)$ .

(e) True or False: There exists a connected graph G with vertices of degrees 3, 3, 4, 4, 4, 4, 6 that contains an Eulerian walk starting at a vertex of degree 4 and ending at a vertex of degree 4.

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