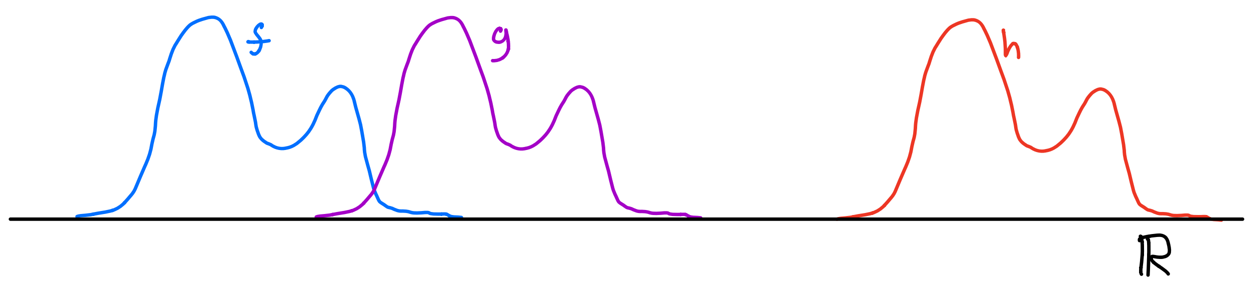
Introduction to the Wasserstein distance

(Also called the Kantorovich-Rubinstein, optimal transport, or earth mover's distance.)



Using function distances, $\|f-g\|_\infty \approx \|f-h\|_\infty$.

Using the Wasserstein distance, $d_W(f,g) < d_W(f,h)$.

What about geodesics?

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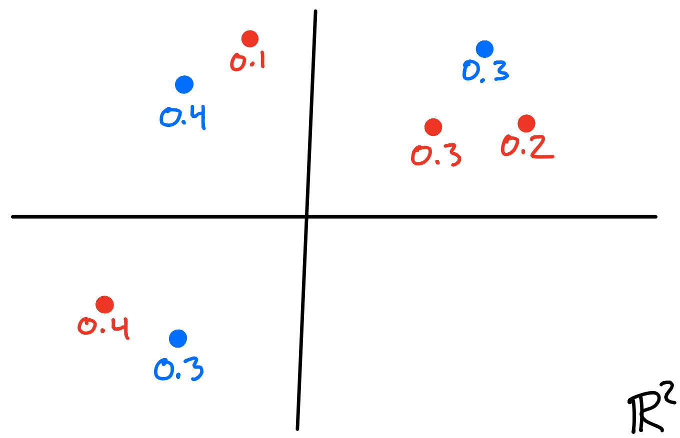

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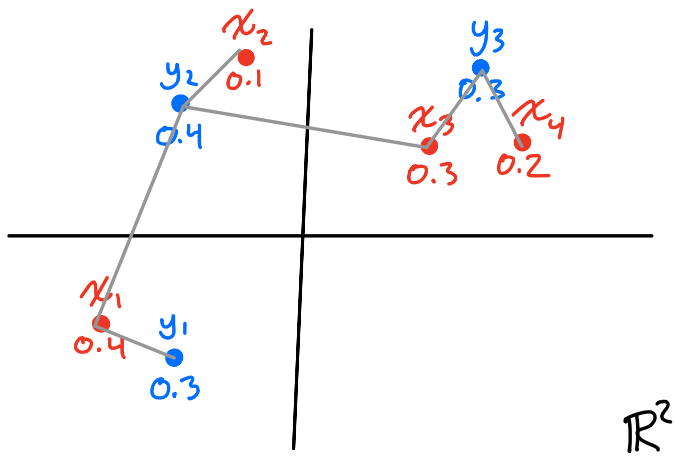

		x_1 0.4	x_2 0.1	x_3 0.3	x_4 0.2
y_1	0.3	0.3	0	0	0
y_2	0.4	0.1	0.1	0.2	0
y_3	0.3	0	0	0.1	0.2

$$d_{W_1}\left(\sum_i \alpha_i \delta_{x_i}, \sum_j \beta_j \delta_{y_j}\right)$$

$$= \min\left\{\sum_{i,j} \pi_{i,j}\, d(x_i, y_j) : \pi_{i,j} \geq 0,\ \sum_i \pi_{i,j} = \beta_j,\ \sum_j \pi_{i,j} = \alpha_i\right\}$$

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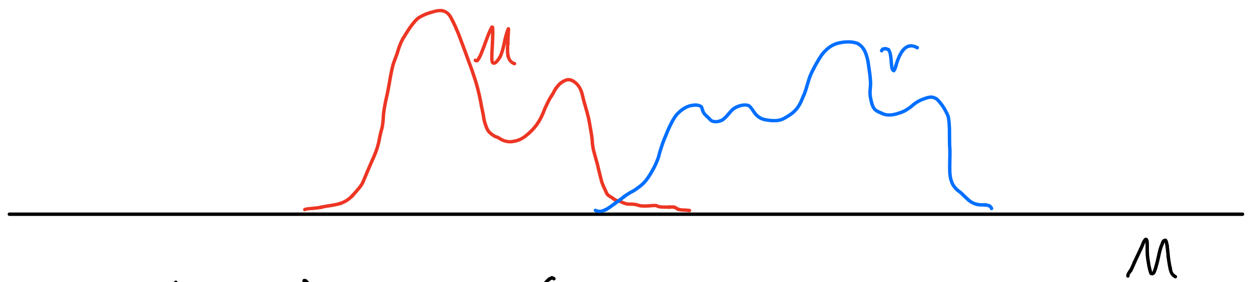

$$d_{W_1}(\mu, \nu) = \inf_{\pi} \int_{M \times M} d(x,y) \, d\pi(x,y)$$

where π is a measure on $M \times M$ with marginals μ and ν.