The unreasonably effective interaction between pure and applied mathematics: A case study on persistence images

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Joint with Sofya Chepushtanova, Tegan Emerson, Eric Hanson, Michael Kirby, Francis Motta, Rachel Neville, Chris Peterson, Patrick Shipman, Lori Ziegelmeier
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"The unreasonable effectiveness of mathematics in the natural sciences"
The mathematical structure of a physical theory points to improved theory and empirical predictions.
Homology (pure) + Data (applied) = Persistent Homology

Topology of cyclo-octane energy landscape
Martin, Thompson, Ounis, Watson, 2010
On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008
Persistent Homology + Machine Learning (applied) = Persistence Images, Draft 1 (among many many other options)

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Persistent Homology + Machine Learning (applied) = Persistence Images, Draft 1 (among many many other options)

Answer: (from left) \( u = 1, 1.75, 2, 1.75, 2, 2 \)
Persistence Homology + Machine Learning (applied) = Persistence Images, Draft 1 (among many many other options)

Diagram $B$ ——— diagram $T(B)$ ——— surface ——— image

Different parameters:

Same parameter:
Persistence Images, Draft 1 + Stability (pure) = Stable Persistence Images
Definition 1 For \( B \) a PD, the corresponding persistence surface \( \rho_B : \mathbb{R}^2 \rightarrow \mathbb{R} \) is the function

\[
\rho_B(z) = \sum_{u \in T(B)} f(u) \phi_u(z).
\]
Definition 1. For $B$ a PD, the corresponding persistence surface $\rho_B : \mathbb{R}^2 \to \mathbb{R}$ is the function

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**Definition 1** For $B$ a PD, the corresponding persistence surface $\rho_B : \mathbb{R}^2 \to \mathbb{R}$ is the function

$$\rho_B(z) = \sum_{u \in T(B)} f(u) \phi_u(z).$$

The number of pure and applied questions then exploded!
On the choice of weight functions for linear representations of persistence diagrams

Vincent Divol$^{1,2}$ · Wolfgang Polonik$^3$
On the choice of weight functions for linear representations of persistence diagrams

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![Weight function diagrams for different values of $n$](https://example.com)

*Fig. 2* For $n = 500$ or $2000$ points uniformly sampled on the torus, persistence images (Adams et al. 2017) for different weight functions are displayed. For $\alpha < 2$, the mass of the topological noise is far larger than the mass of the true signal, the latter being comprised by the two points with high-persistence. For $\alpha = 2$, the two points with high-persistence are clearly distinguishable. For $\alpha = 100$, the noise has also disappeared, but so has one of the points with high-persistence.
Persistance diagrams with linear machine learning models

Ippei Obayashi¹ · Yasuaki Hiraoka²,³,⁴ · Masao Kimura⁵,⁶

Fig. 4 Input binary images and their 0th persistence diagrams. The left and right two images are sampled from the parameter pairs (A) and (B), respectively.
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Fig. 5  a The reconstructed persistence diagram from the learned vector w. The blue (resp. red) area contributes to the class 0 (resp. 1). b A thresholding of (a). c 1–4 The birth positions of the generators in blue and red areas in (b) are plotted with the same color (color figure online)
Topology Applied to Machine Learning: From Global to Local

Henry Adams and Michael Moy*
Local geometry

Measures of order for nearly hexagonal lattices
Motta, Neville, Shipman, Pearson, Bradley, 2018
Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015
Local geometry

Analysis of Kolmogorov flow and Rayleigh–Bénard convection using persistent homology
Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischaikow, 2016
Local geometry

Understanding diffusion patterns of glassy, liquid and amorphous materials via persistent homology analysis
Onodera, Kohara, Tahara, Masuno, Inoue, Shiga, Hirata, Tsuchiya, Hirooka, Obayashi, Ohara, Mizuno, Sakata, 2019
Local geometry

Hyperbolic disk  Flat disk  Disk on sphere

Persistent homology detects curvature
Bubenik, Hull, Patel, Whittle, 2019
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

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Further progress in a series of papers
by Benjamin Schweinhart
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