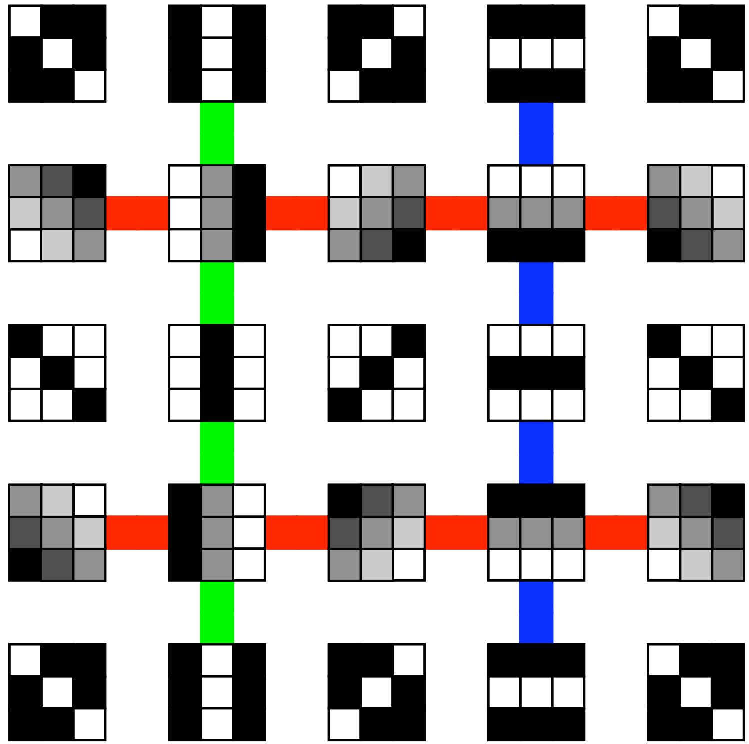
Applied Topology



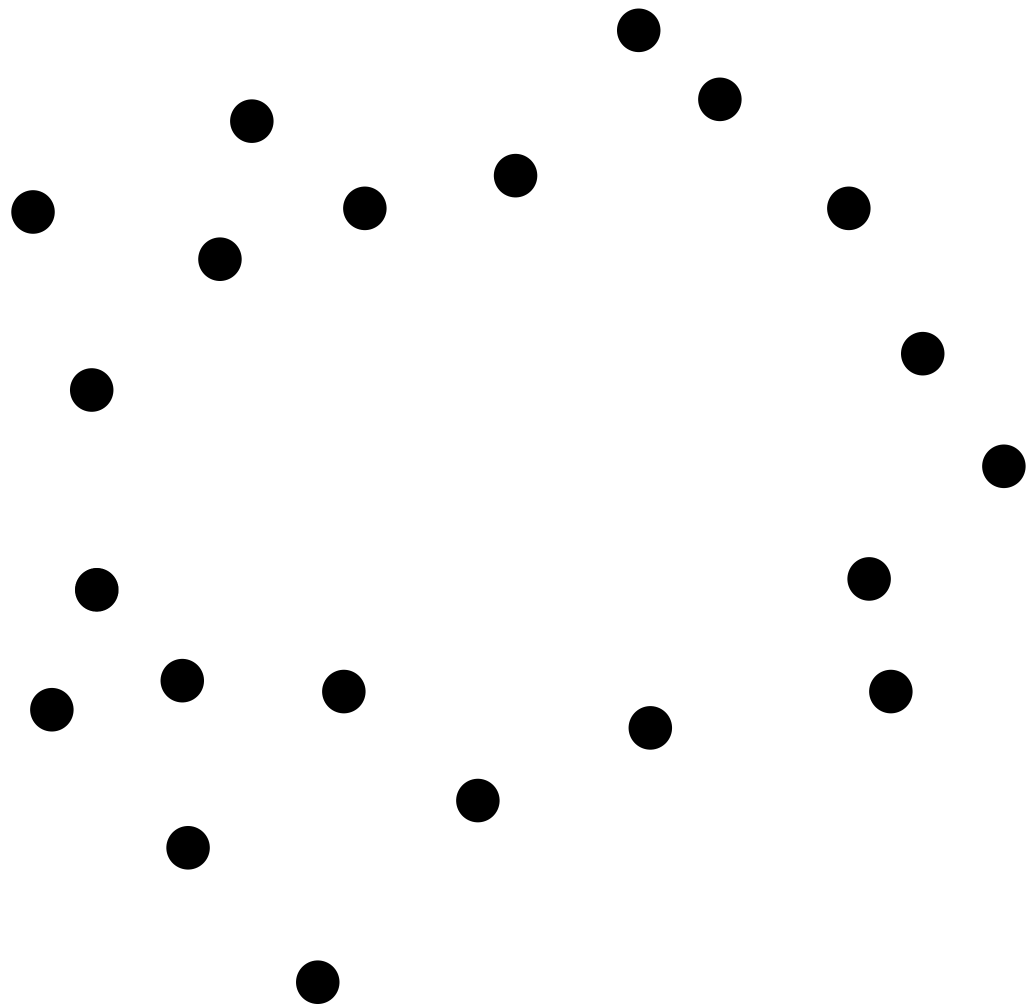
Henry Adams

July 18, 2013

Stanford University Mathematics Camp

Datasets have shapes

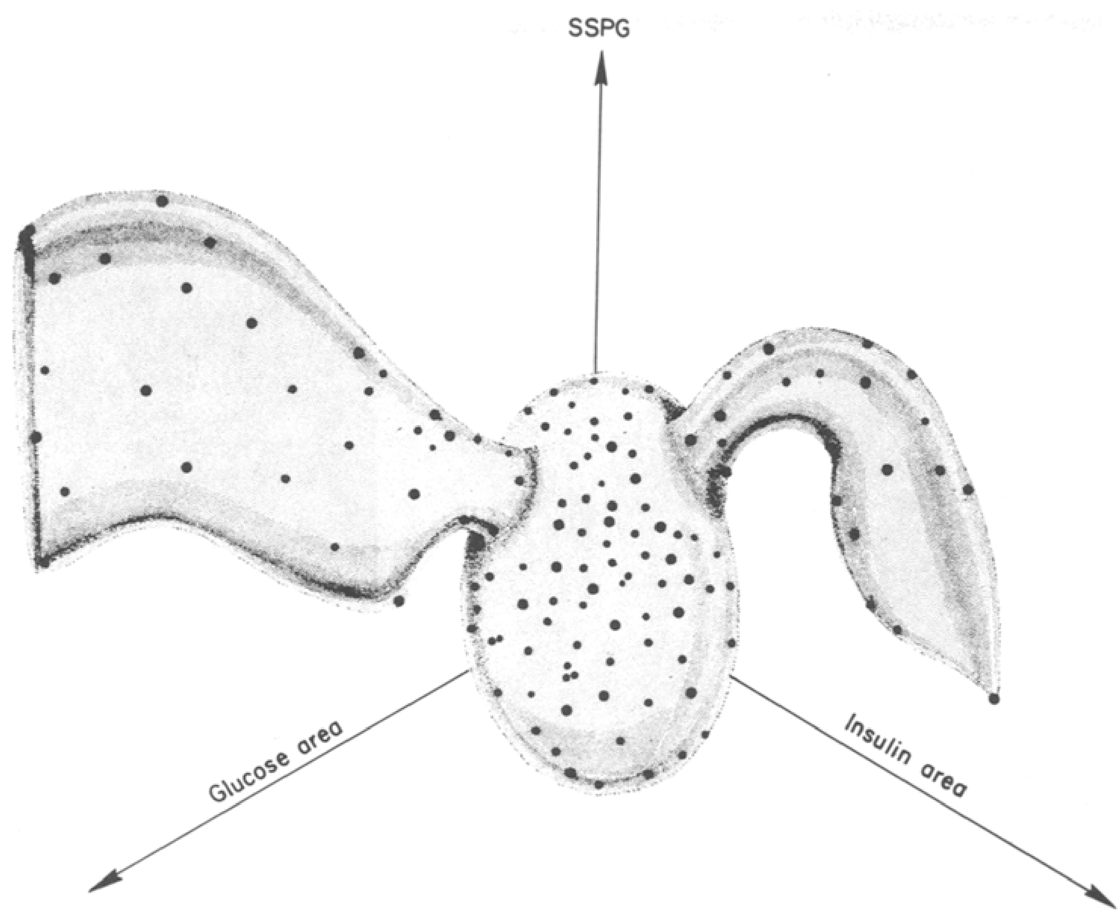
What shape is this?



Datasets have shapes

Example: Diabetes study

145 points in 5-dimensional space



An Attempt to Define the Nature of Chemical Diabetes Using a Multidimensional Analysis by G. M. Reaven and R. G. Miller, 1979.

Datasets have shapes

Example: Cyclo-Octane (C_8H_{16}) data

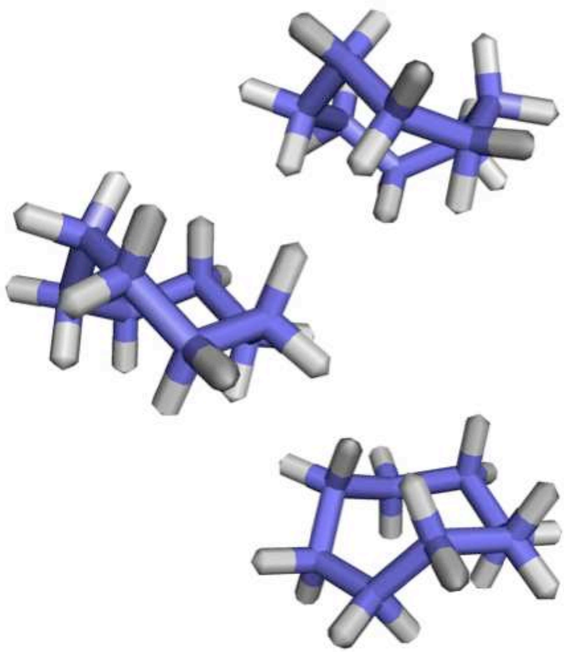
1,031,644 points in 72-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data by Shawn Martin and Jean-Paul Watson, 2010.

Datasets have shapes

Example: Cyclo-Octane (C_8H_{16}) data

1,031,644 points in 72-dimensional space



$$\Longrightarrow \underbrace{\begin{bmatrix} c_{1,1,x} & c_{2,1,x} & \\ c_{1,1,y} & c_{2,1,y} & \\ c_{1,1,z} & c_{2,1,z} & \\ \vdots & \vdots & \cdots \\ h_{1,16,x} & h_{2,16,x} & \\ h_{1,16,y} & h_{2,16,y} & \\ h_{1,16,z} & h_{2,16,z} & \end{bmatrix}}_{1,031,644} \left.\vphantom{\begin{bmatrix} a\\a\\a\\a\\a\\a\\a \end{bmatrix}}\right\} 72 \Longrightarrow$$

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Datasets have shapes

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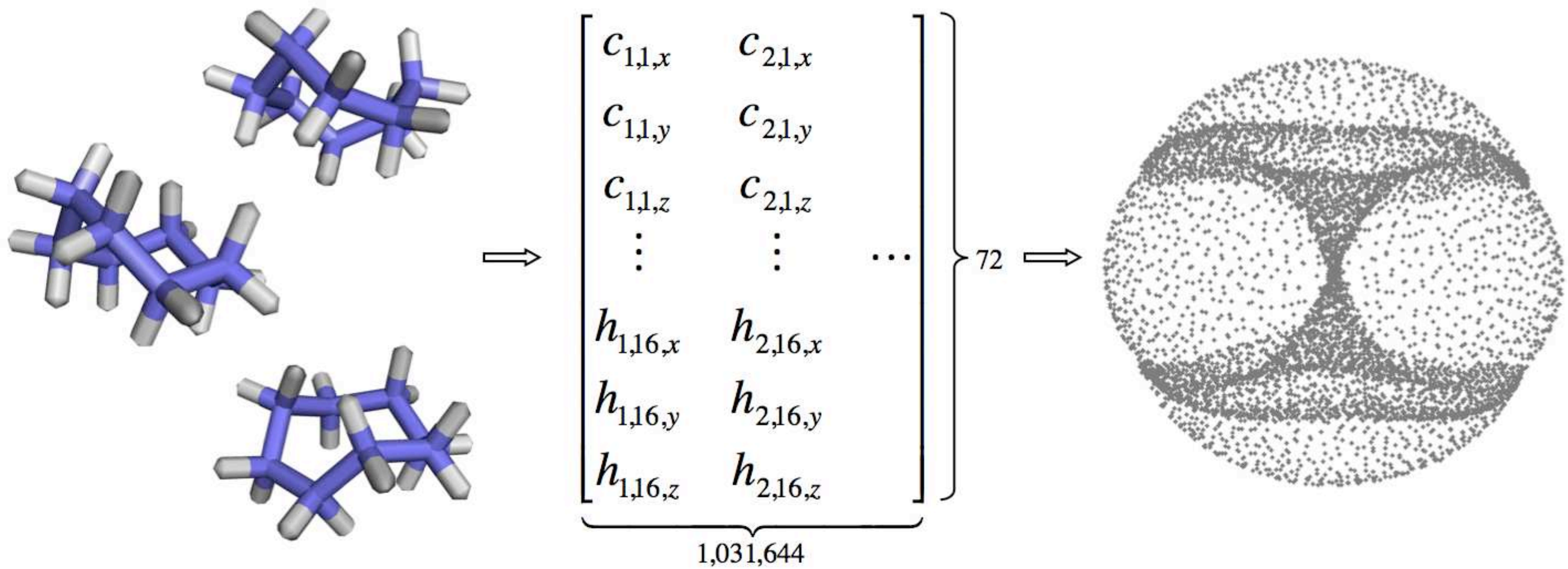

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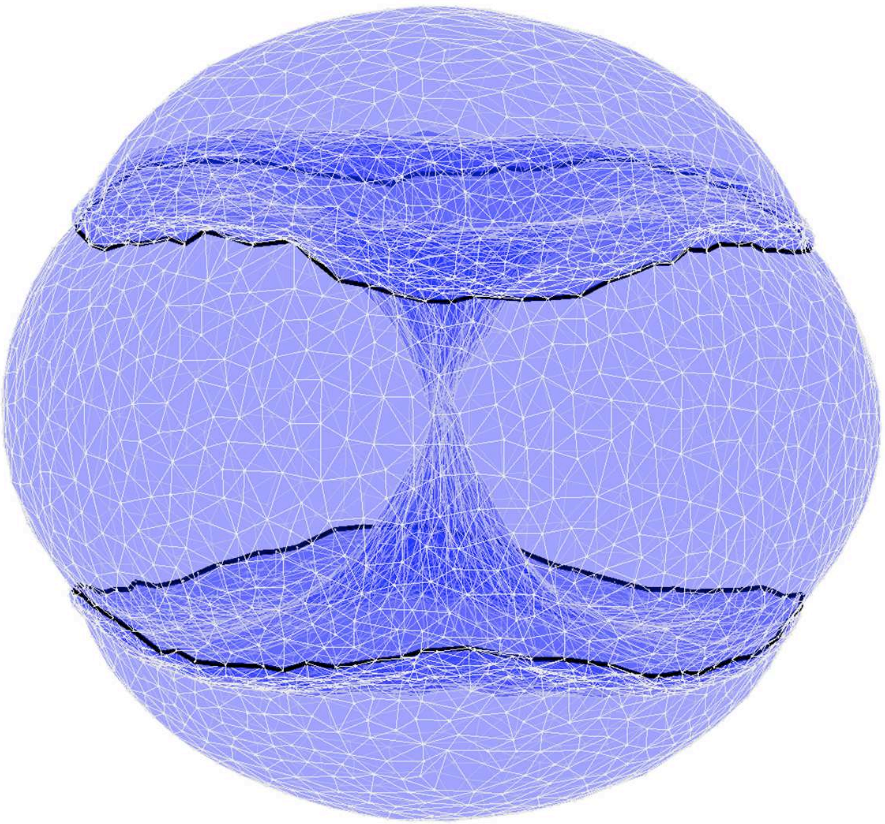

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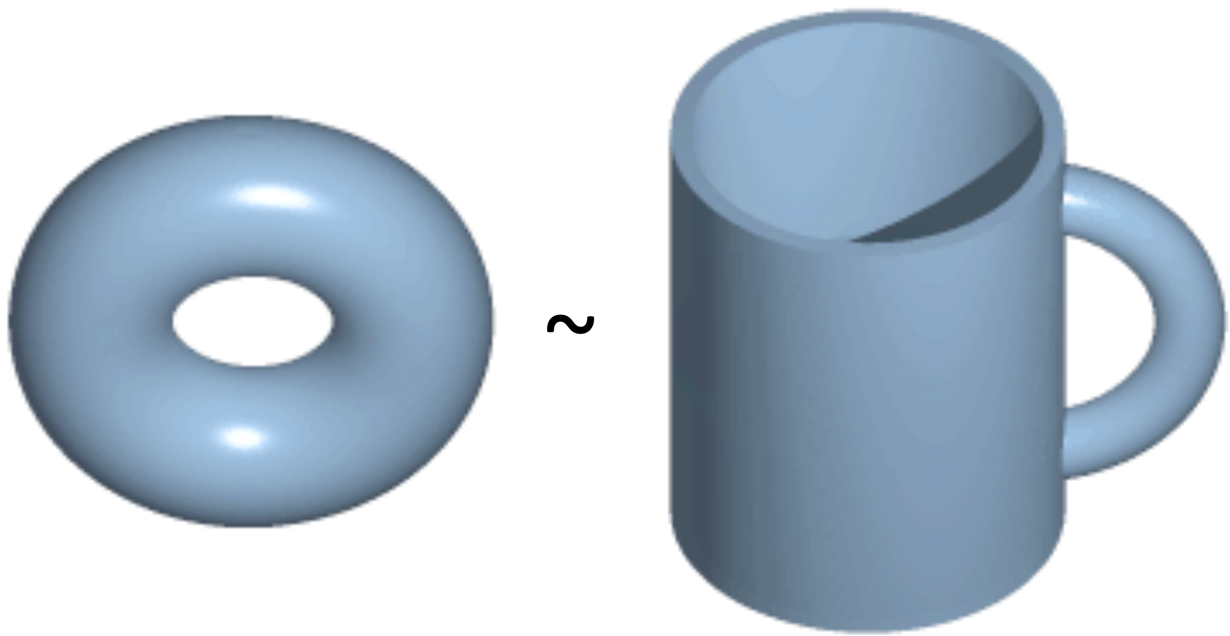
Topology studies shapes

A donut and coffee mug are “homotopy equivalent.”



Topology studies shapes

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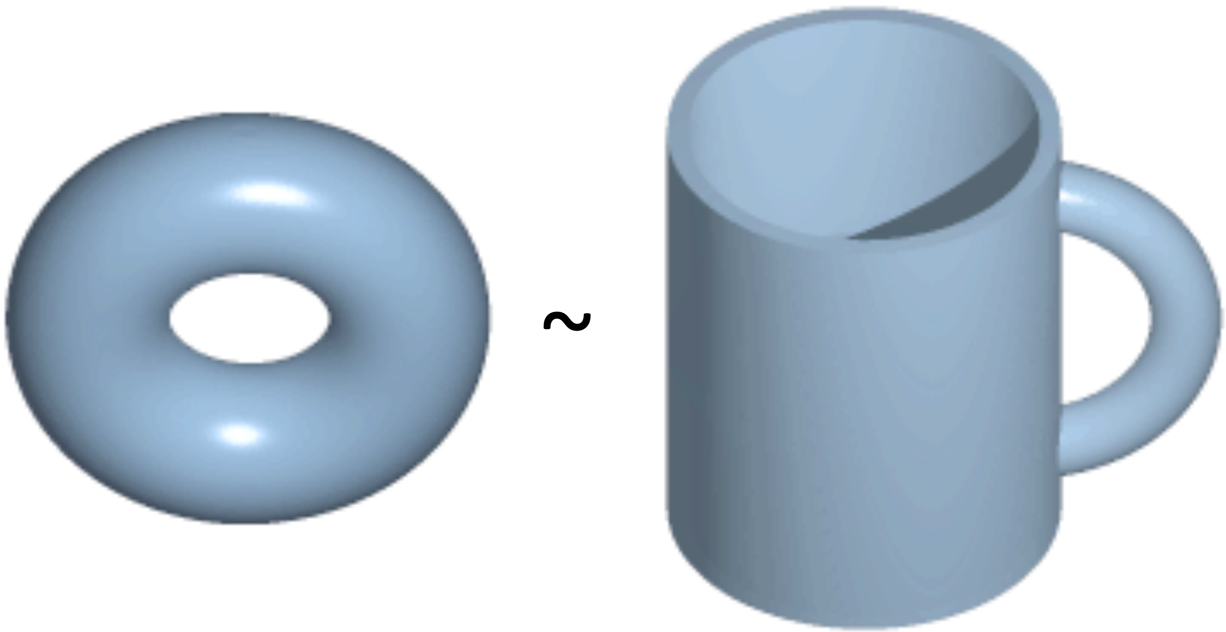
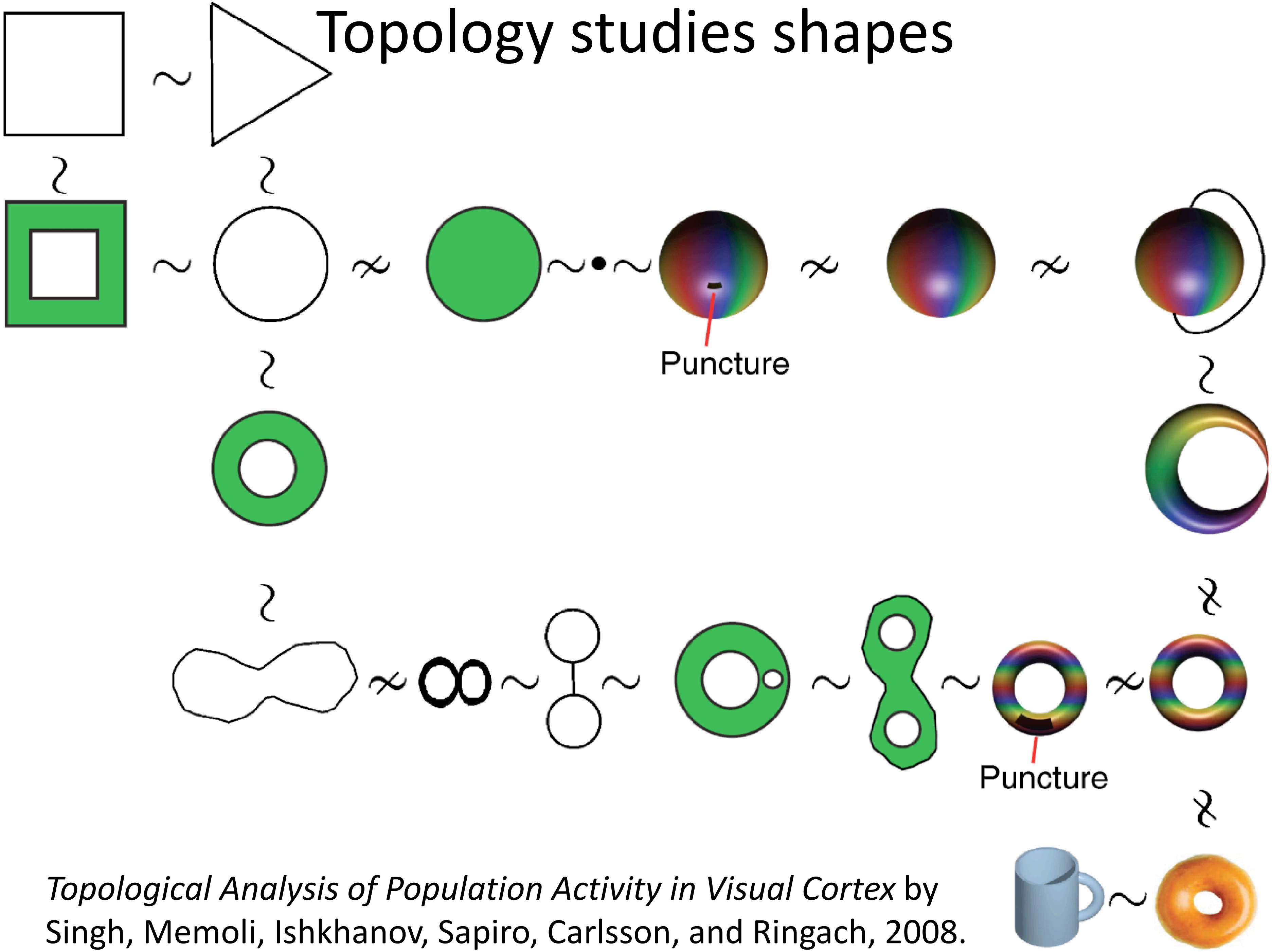


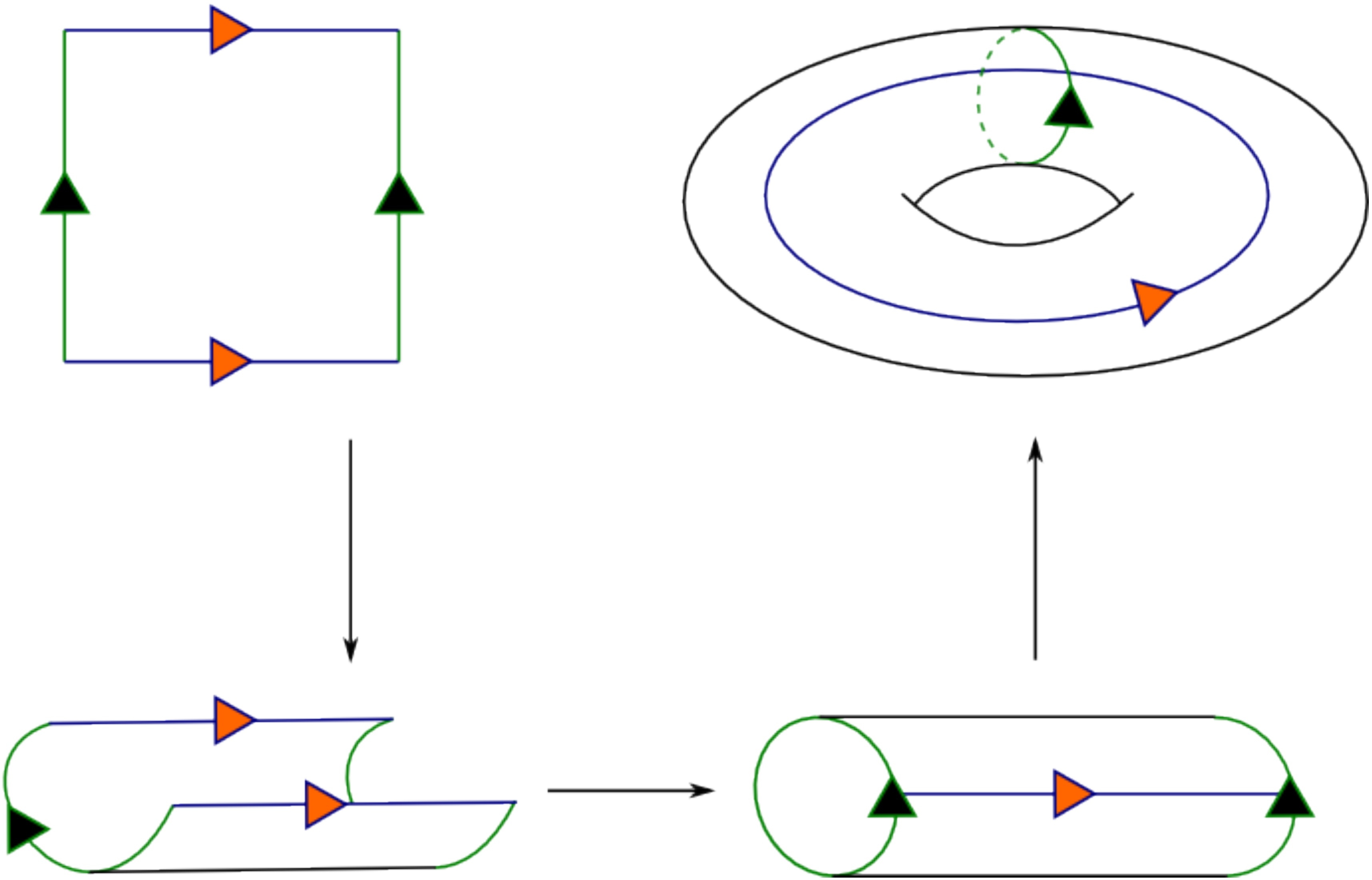
Topology studies shapes



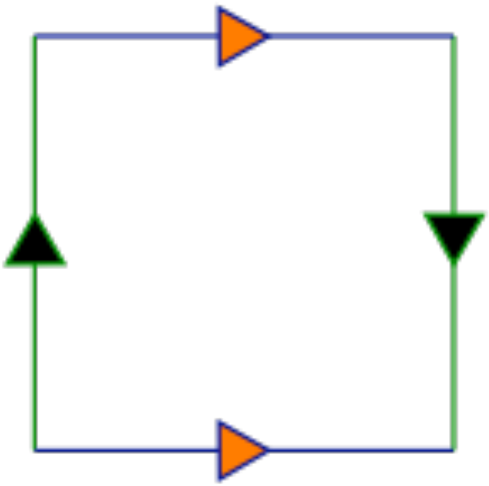
Topological Analysis of Population Activity in Visual Cortex by Singh, Memoli, Ishkhanov, Sapiro, Carlsson, and Ringach, 2008.

Topology studies shapes

Torus

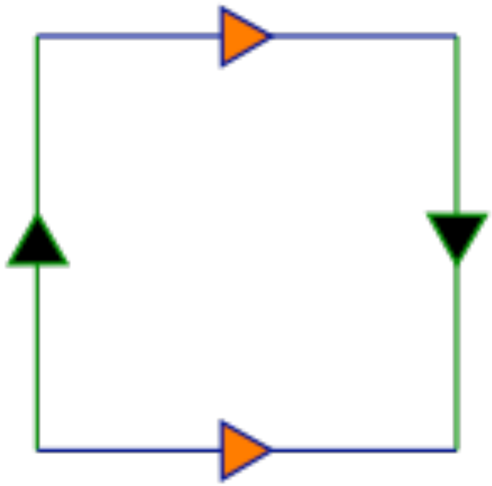


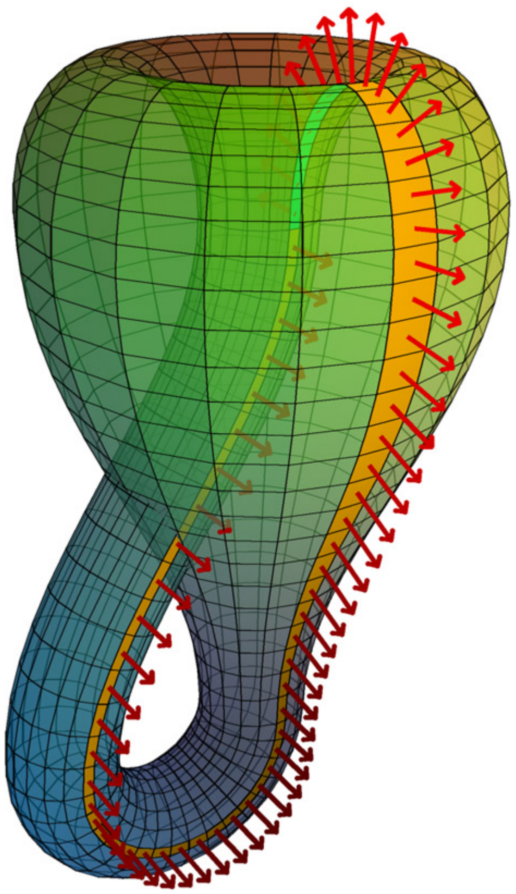
Topology studies shapes



Topology studies shapes

Klein bottle





https://plus.maths.org/content/imaging-maths-inside-klein-bottle

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

$$
\begin{tikzcd}
 & \vdots \arrow[d] & \vdots \arrow[d] & \vdots \arrow[d] & \\
0 \arrow[r] & A_{n+1} \arrow[r, "\alpha_{n+1}"] \arrow[d, "\partial_{n+1}"'] & B_{n+1} \arrow[r, "\beta_{n+1}"] \arrow[d, "\partial'_{n+1}"'] & C_{n+1} \arrow[r] \arrow[d, "\partial''_{n+1}"'] & 0 \\
0 \arrow[r] & A_{n} \arrow[r, "\alpha_{n}"] \arrow[d, "\partial_{n}"'] & B_{n} \arrow[r, "\beta_{n}"] \arrow[d, "\partial'_{n}"'] & C_{n} \arrow[r] \arrow[d, "\partial''_{n}"'] & 0 \\
0 \arrow[r] & A_{n-1} \arrow[r, "\alpha_{n-1}"] \arrow[d] & B_{n-1} \arrow[r, "\beta_{n-1}"] \arrow[d] & C_{n-1} \arrow[r] \arrow[d] & 0 \\
 & \vdots & \vdots & \vdots &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & & \vdots \arrow[dll] \\
H_{n+1}(\mathcal{A}) \arrow[r, "\alpha_*"'] & H_{n+1}(\mathcal{B}) \arrow[r, "\beta_*"] & H_{n+1}(\mathcal{C}) \arrow[dll, "\delta_{n+1}"] \\
H_{n}(\mathcal{A}) \arrow[r, "\alpha_*"'] & H_{n}(\mathcal{B}) \arrow[r, "\beta_*"] & H_{n}(\mathcal{C}) \arrow[dll, "\delta_{n}"] \\
H_{n-1}(\mathcal{A}) \arrow[r, "\alpha_*"'] & H_{n-1}(\mathcal{B}) \arrow[r, "\beta_*"] & H_{n-1}(\mathcal{C}) \arrow[dl] \\
 & \vdots &
\end{tikzcd}
$$

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

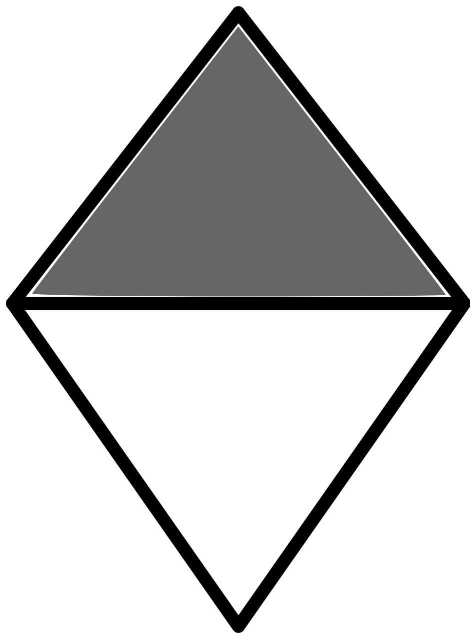
Homology groups H_0, H_1, H_2, H_3, …

H_k “counts the number of k-dimensional holes”.

Homotopy equivalent shapes have the same homology groups.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



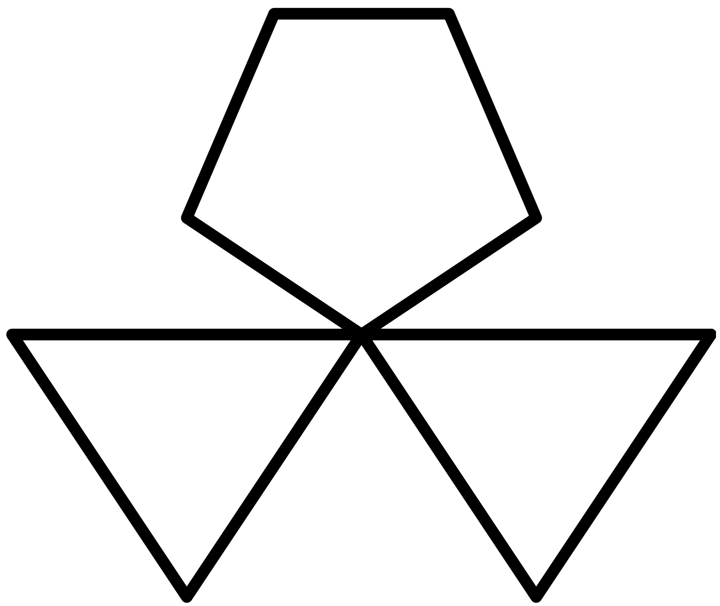
H_0 has rank 1.

H_1 has rank 1.

H_2 has rank 0.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



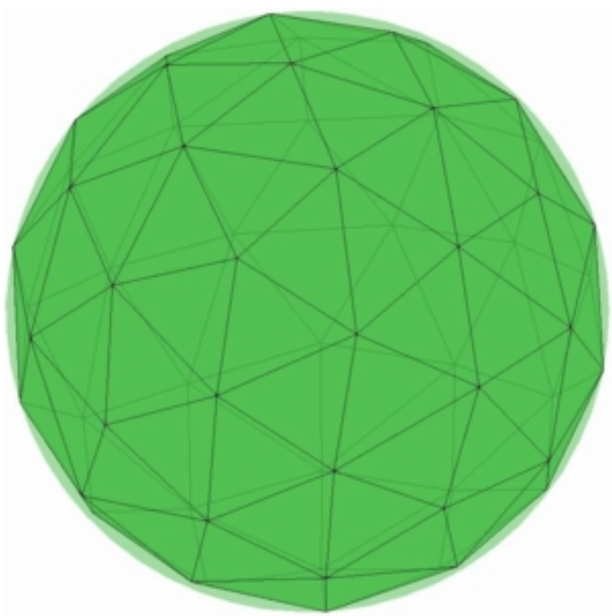
H_0 has rank 1.

H_1 has rank 3.

H_2 has rank 0.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



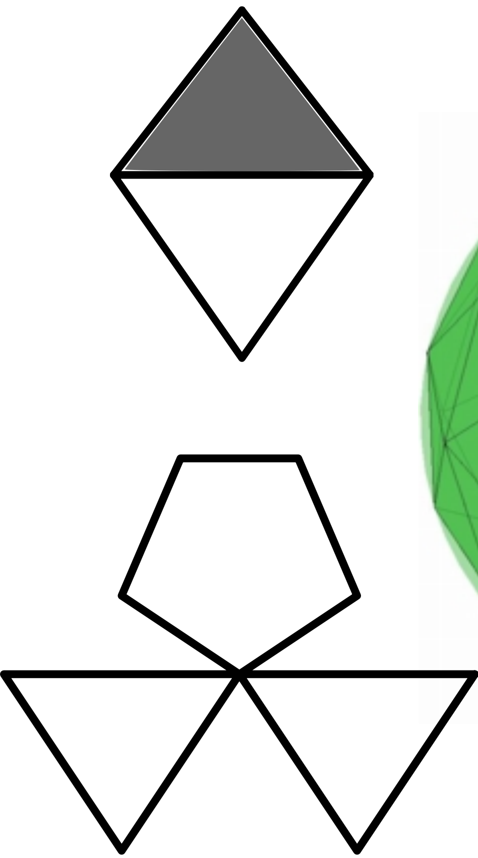
H_0 has rank 1.

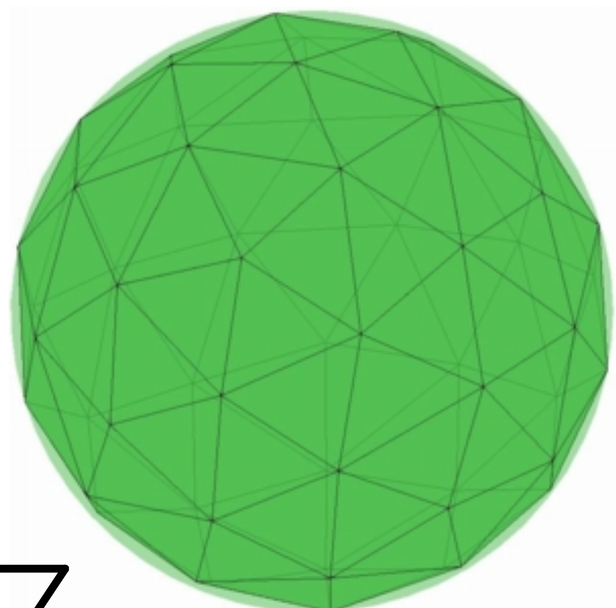
H_1 has rank 0.

H_2 has rank 1.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)





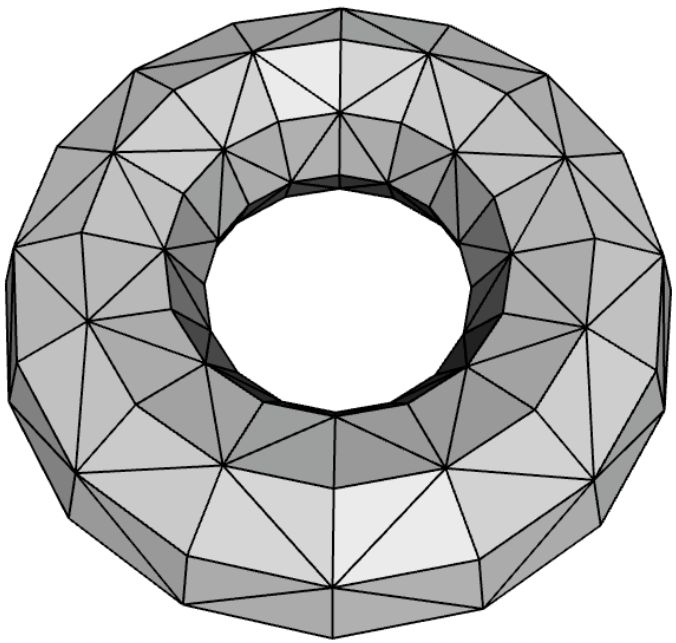
H_0 has rank 3.

H_1 has rank 4.

H_2 has rank 1.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



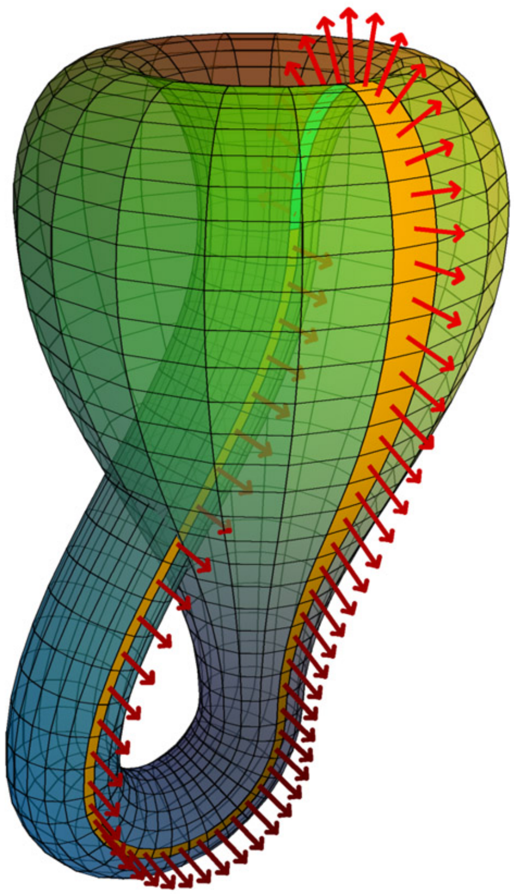
H_0 has rank 1.

H_1 has rank 2.

H_2 has rank 1.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



H_0 has rank 1.

H_1 has rank 2.

H_2 has rank 1.

https://plus.maths.org/content/imaging-maths-inside-klein-bottle

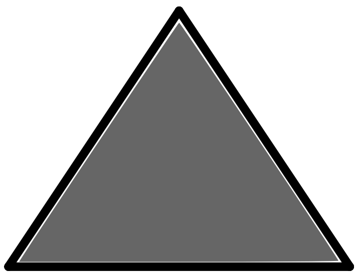
Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

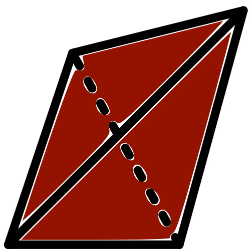
0-simplex

1-simplex

2-simplex



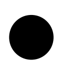
3-simplex



Topology studies shapes

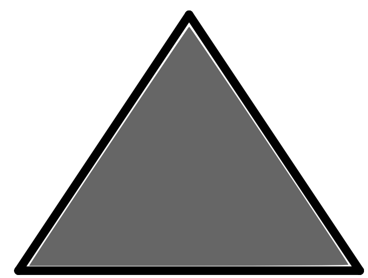
Homology ($\mathbb{Z}/2\mathbb{Z}$)

0-simplex

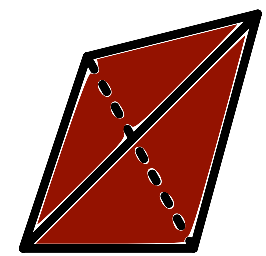


1-simplex

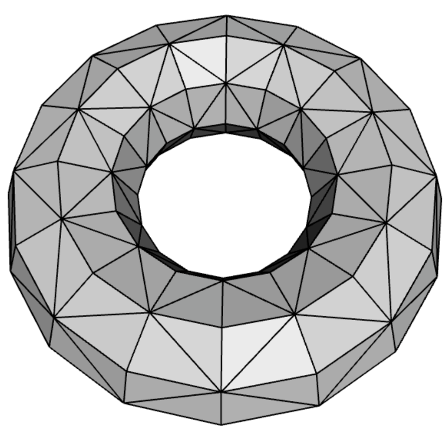
2-simplex

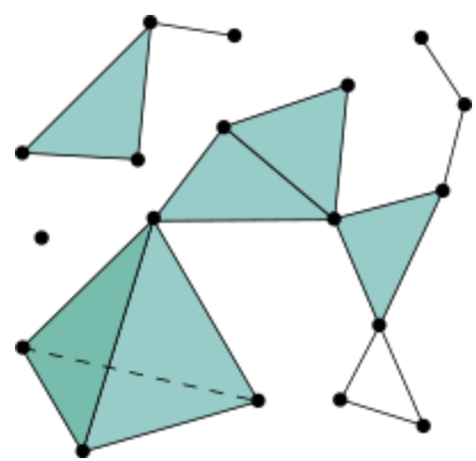


3-simplex



Simplicial complexes





Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

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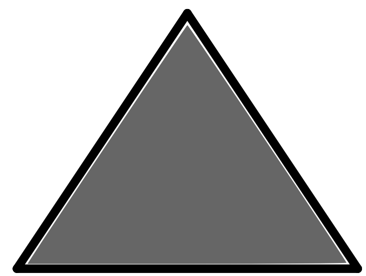
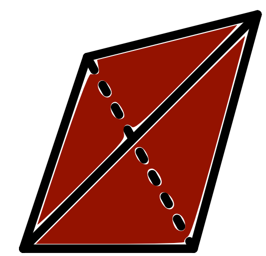
1-simplex

2-simplex



3-simplex



Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

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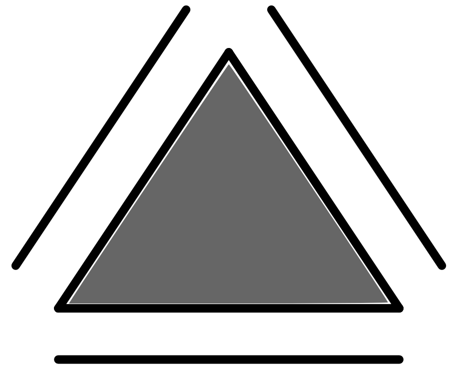
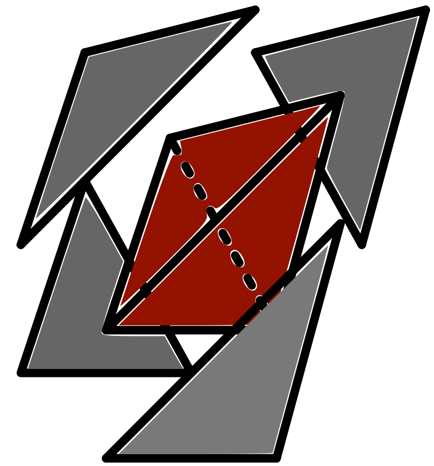
1-simplex

2-simplex



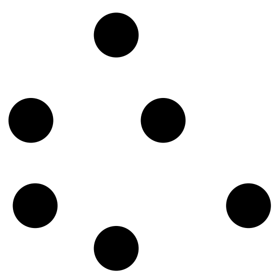
3-simplex



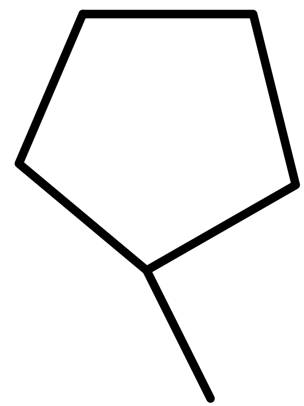
Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

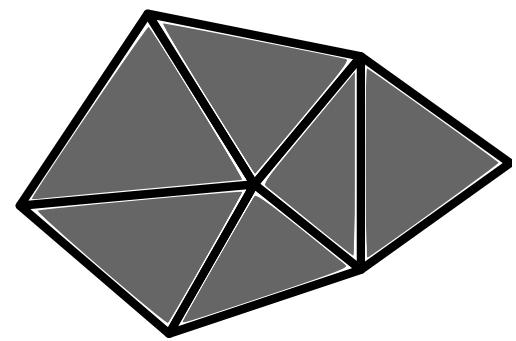
0-simplices



1-simplices



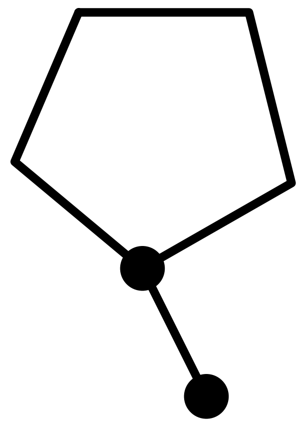
2-simplices

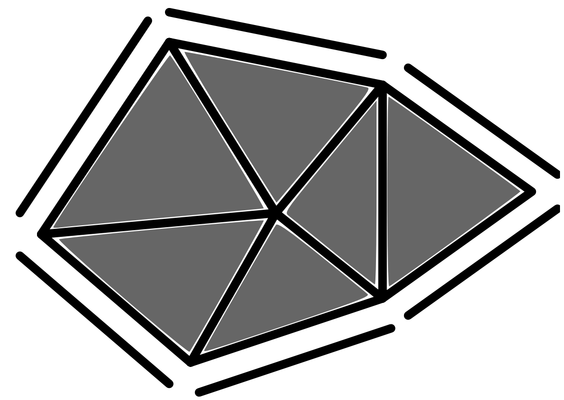


Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

0-simplices 1-simplices 2-simplices

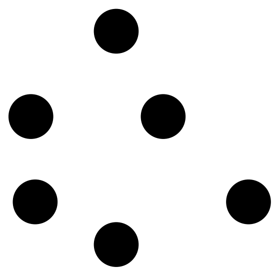




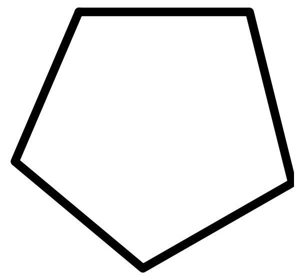
Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

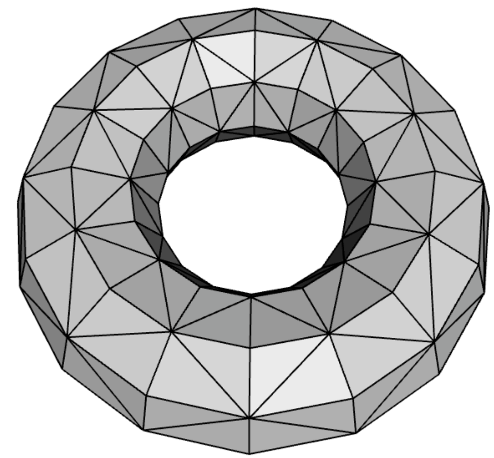
0-cycle



1-cycle



2-cycle

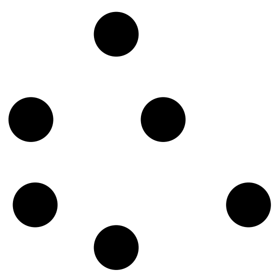


A cycle has no boundary.

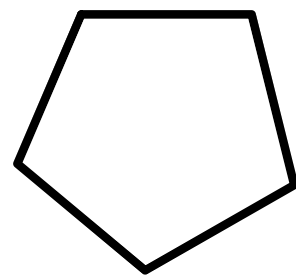
Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

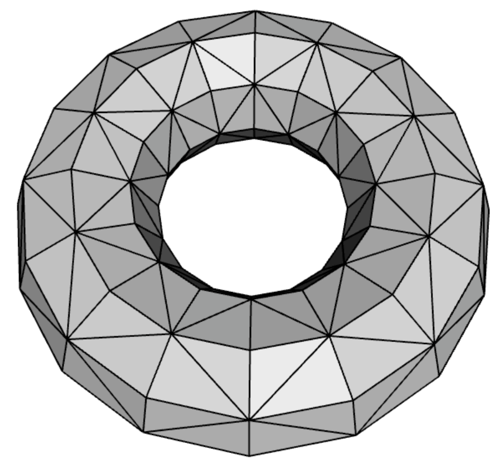
0-cycle



1-cycle



2-cycle



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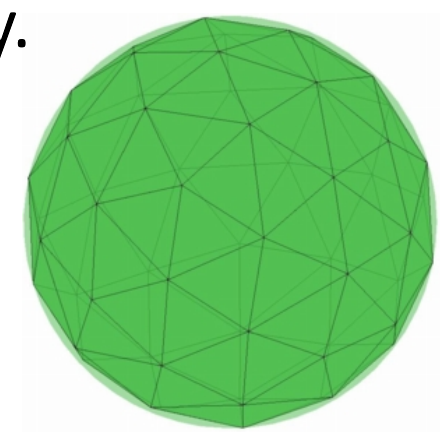
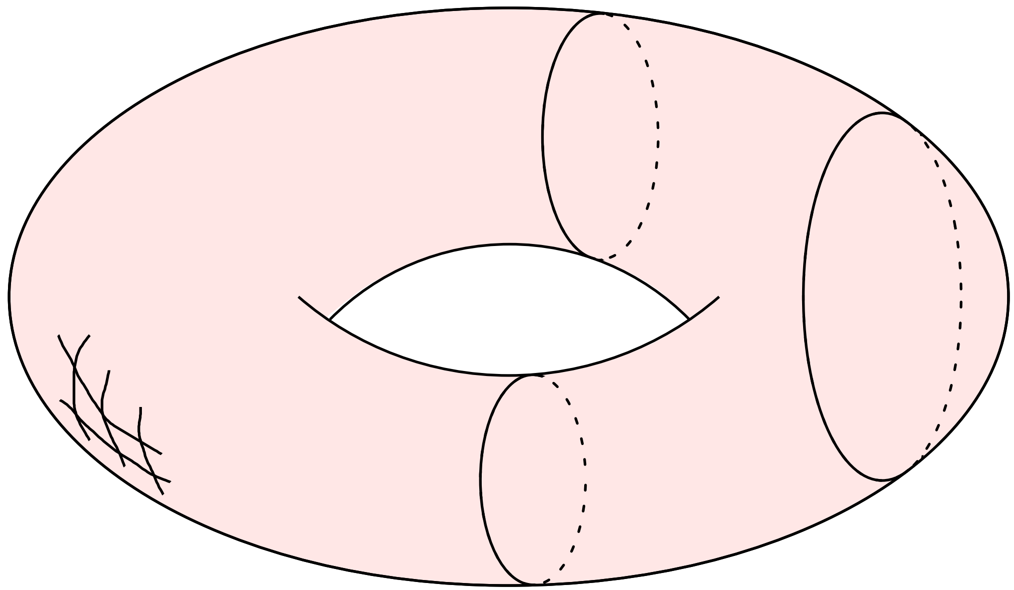


Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

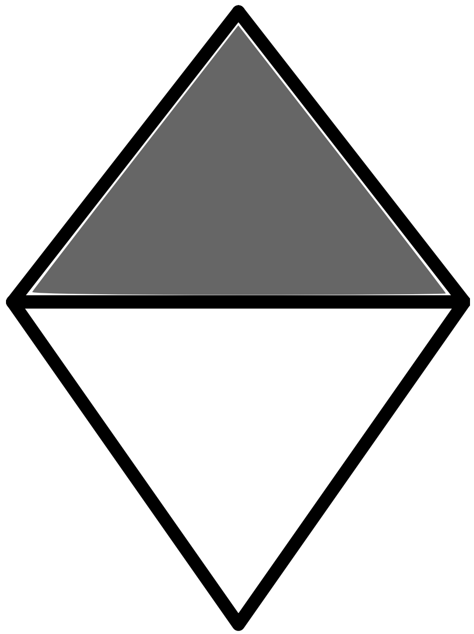


Two cycles are equivalent if they differ by a boundary.

H_k measures equivalence classes of k-cycles.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



H_0 has rank 1.

H_1 has rank 1.

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Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

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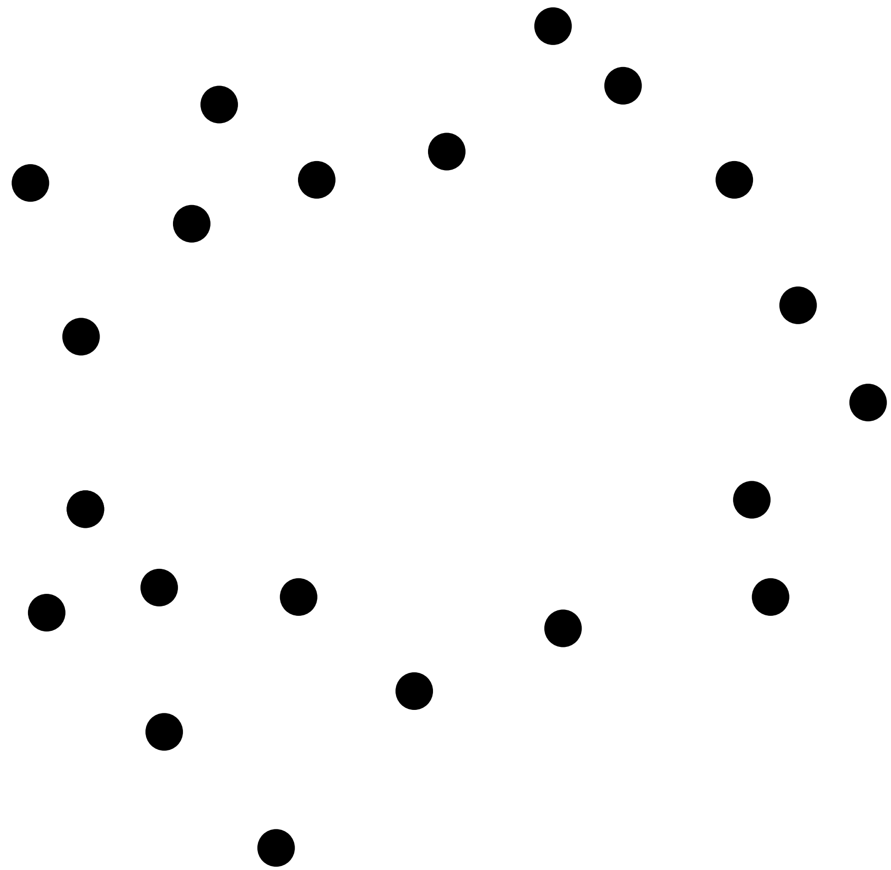
"Topology! The stratosphere of human thought! In the twenty-fourth century it might possibly be of use to someone …"

-Aleksandr Solzhenitsyn, *The First Circle*

Topology applied to data analysis

What shape is this?



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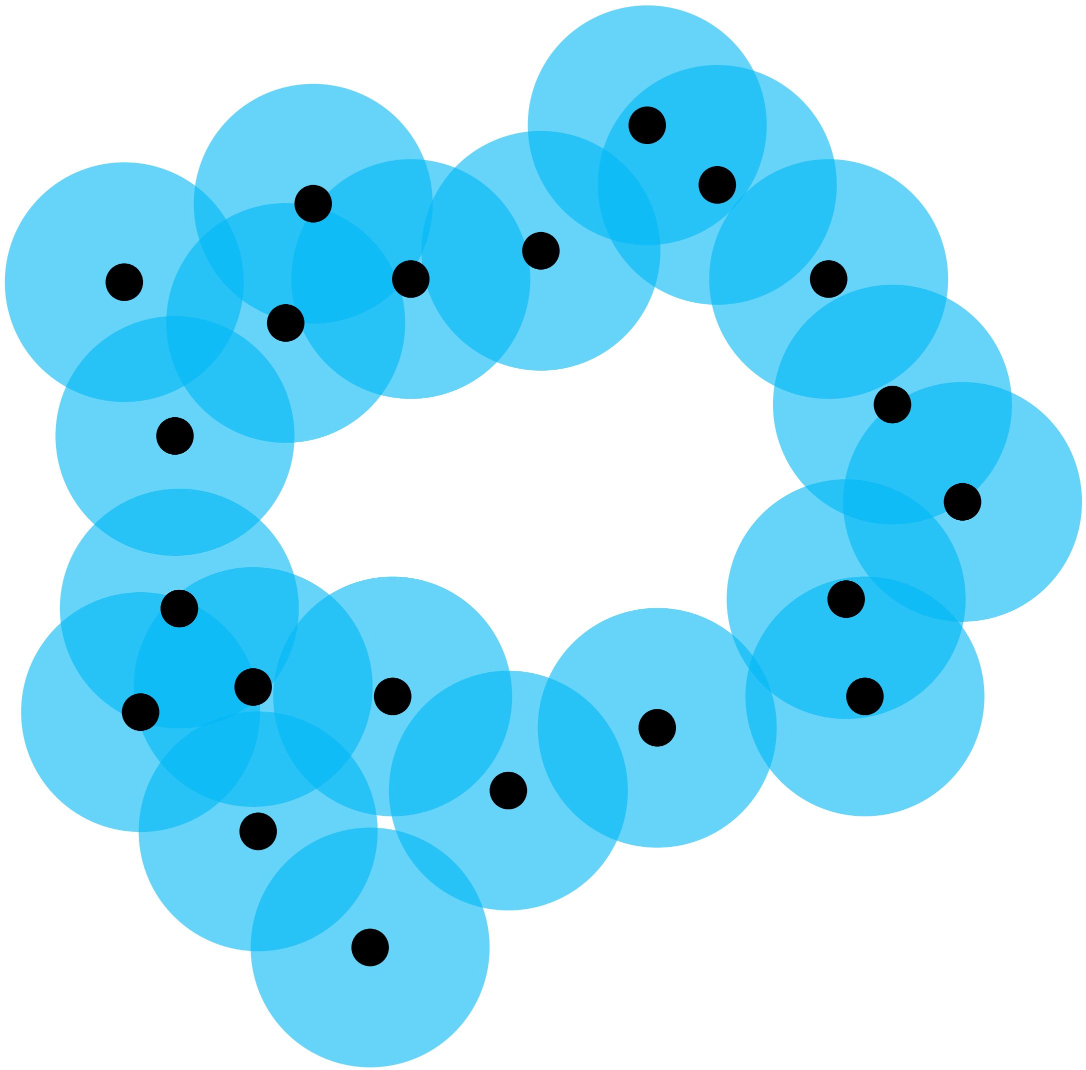
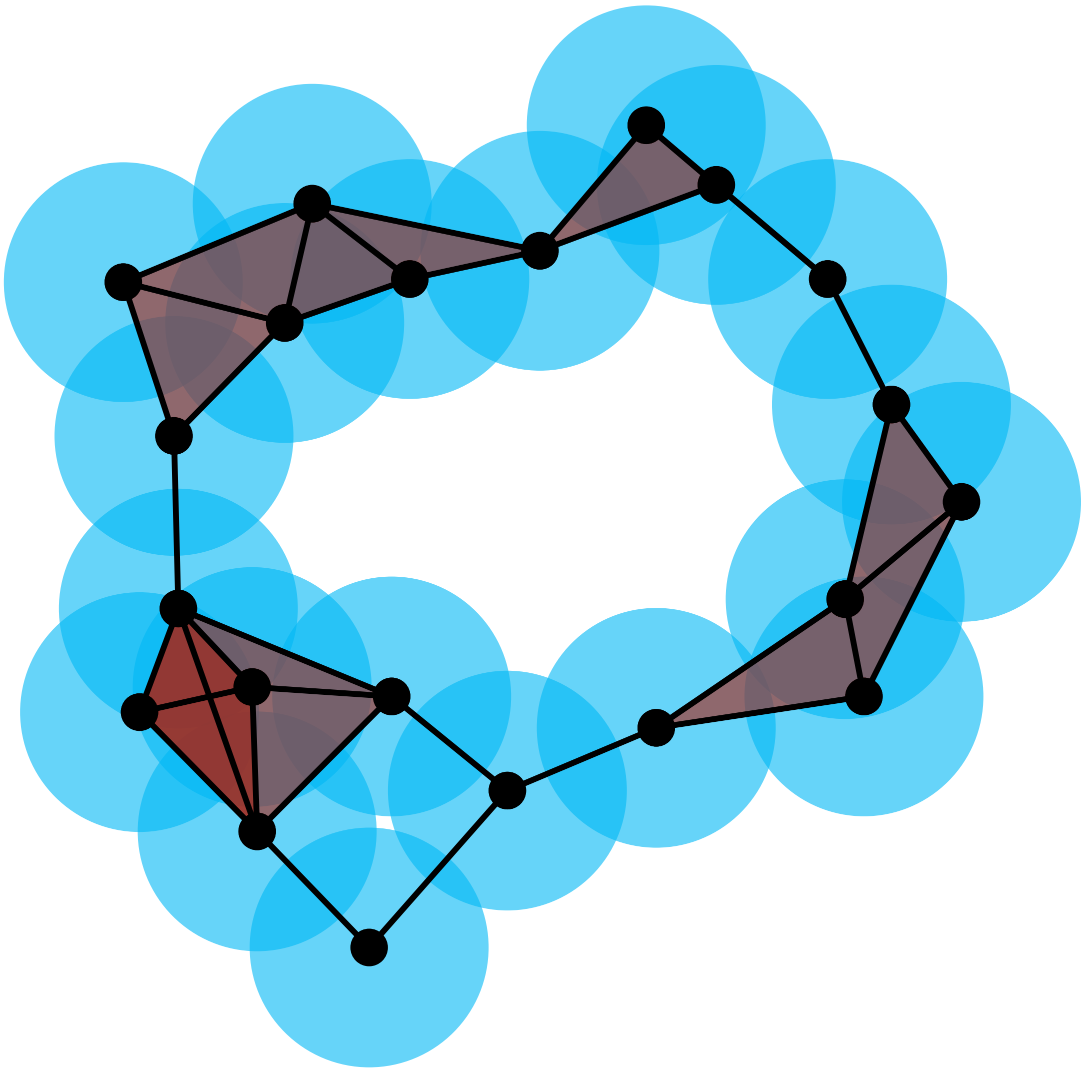


Topology applied to data analysis

What shape is this?

Cech complex



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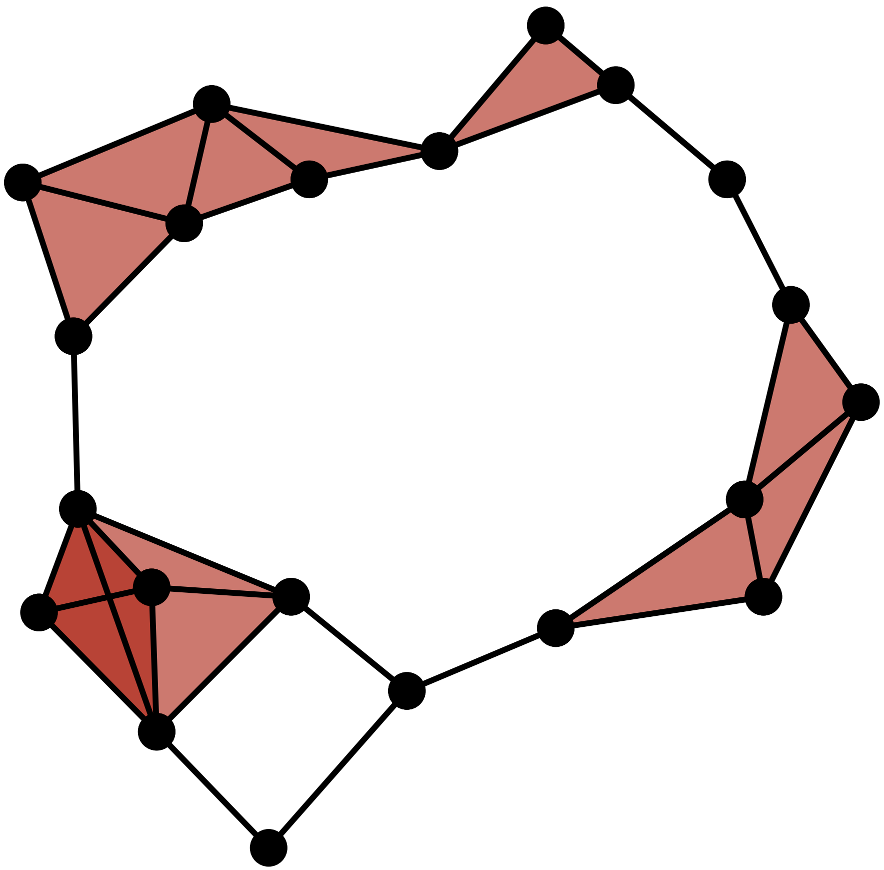

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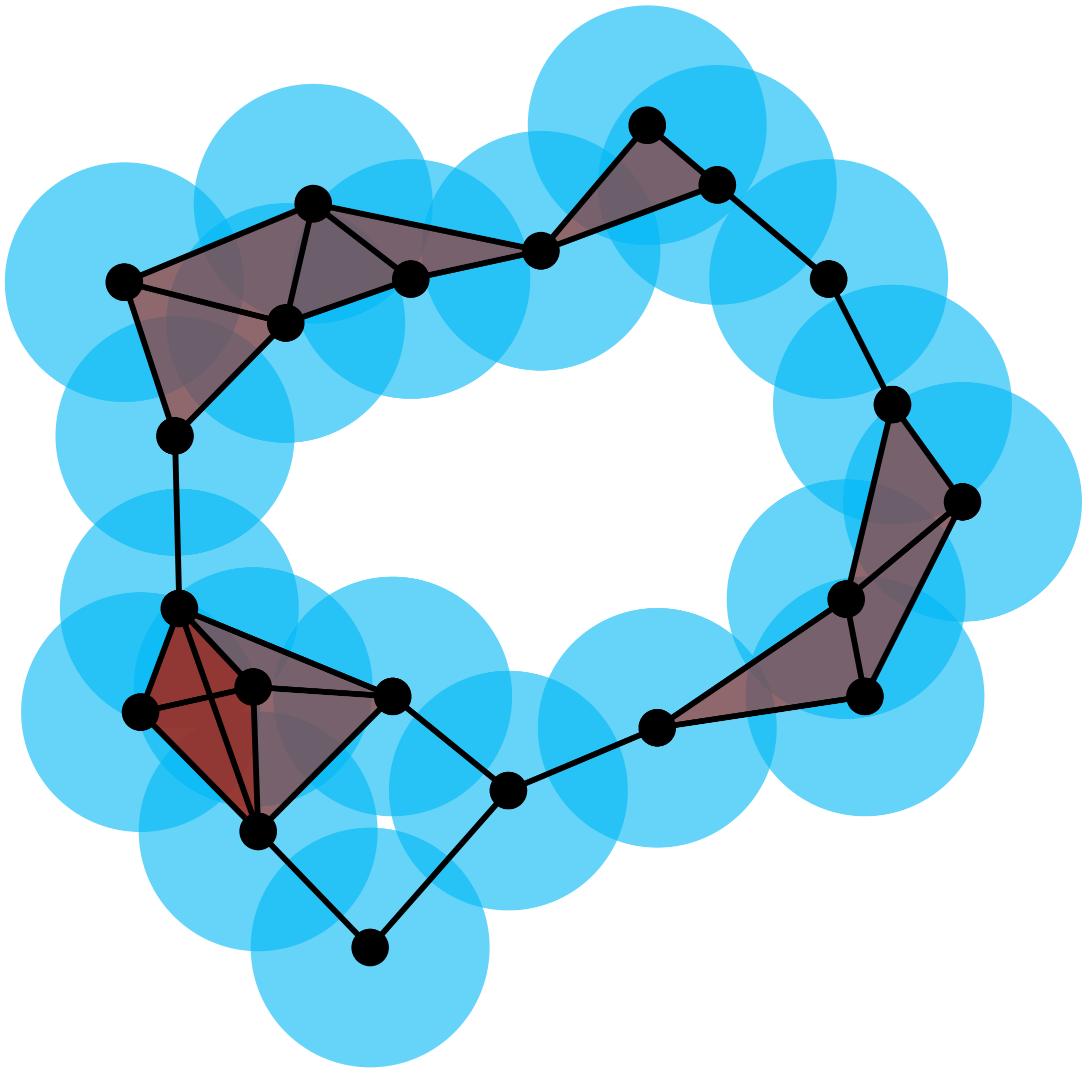

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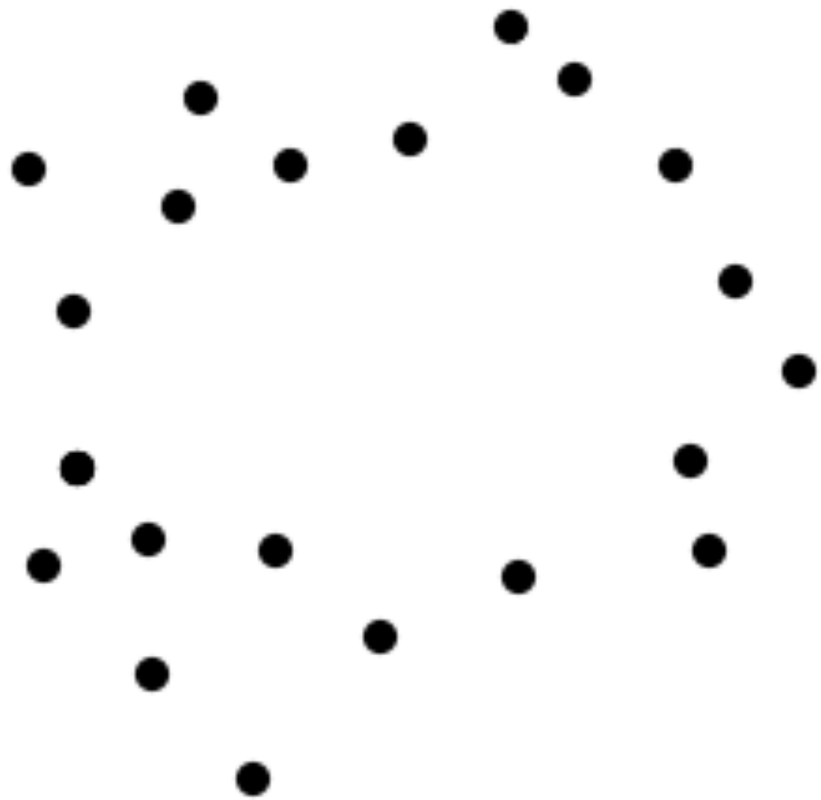

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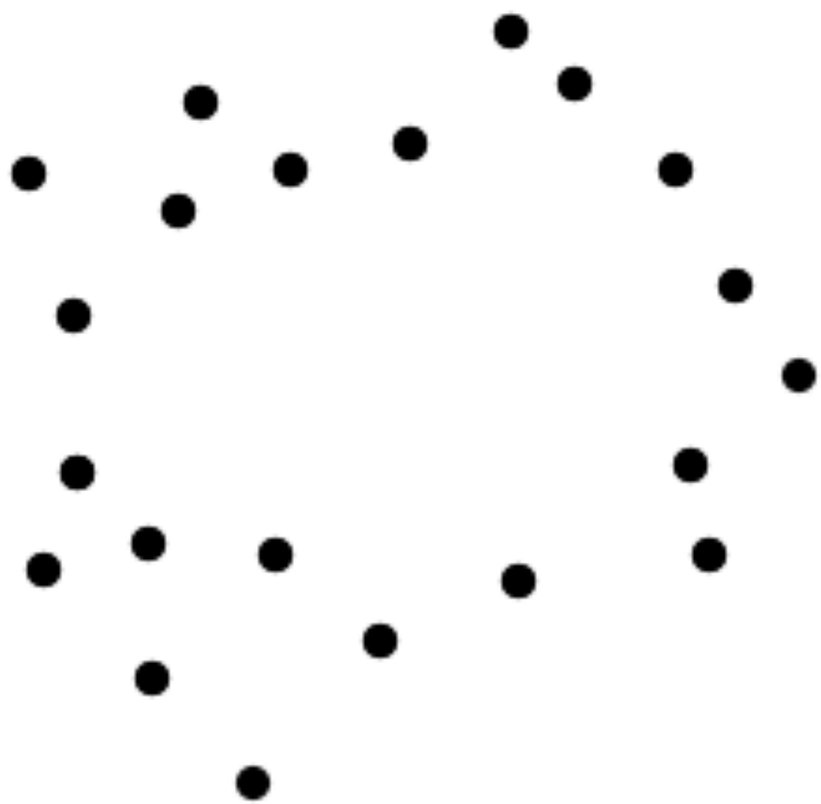
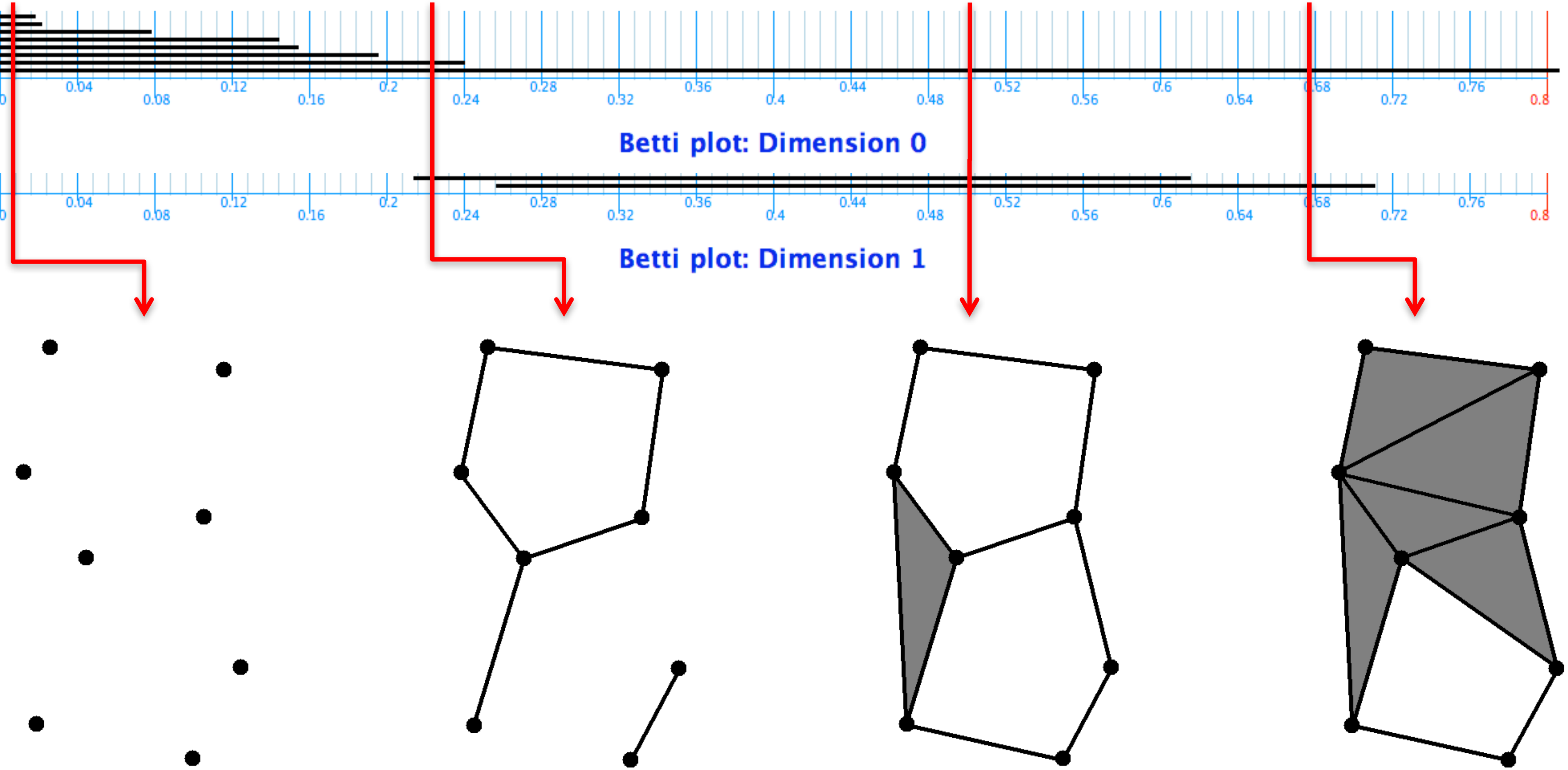


Topology applied to data analysis

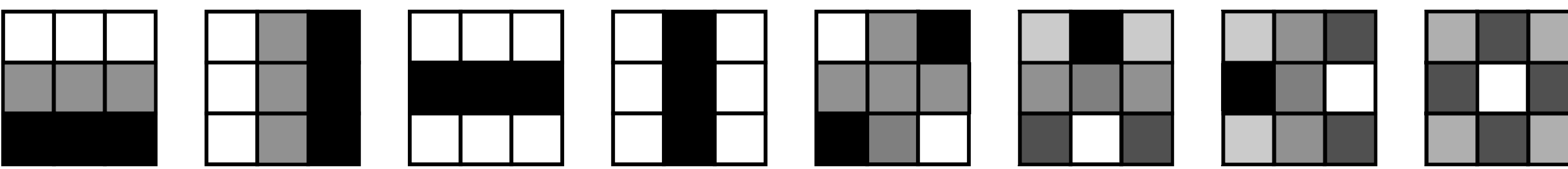


Significant features persist.

Image patch example

Study 3x3 high-contrast patches from images

Points in 9-dimensional space



On the Local Behavior of Spaces of Natural Images by Gunnar Carlsson, Tigran Ishkhanov, Vin de Silva, and Afra Zomorodian, 2008.

Image patch example

1st densest group of patches

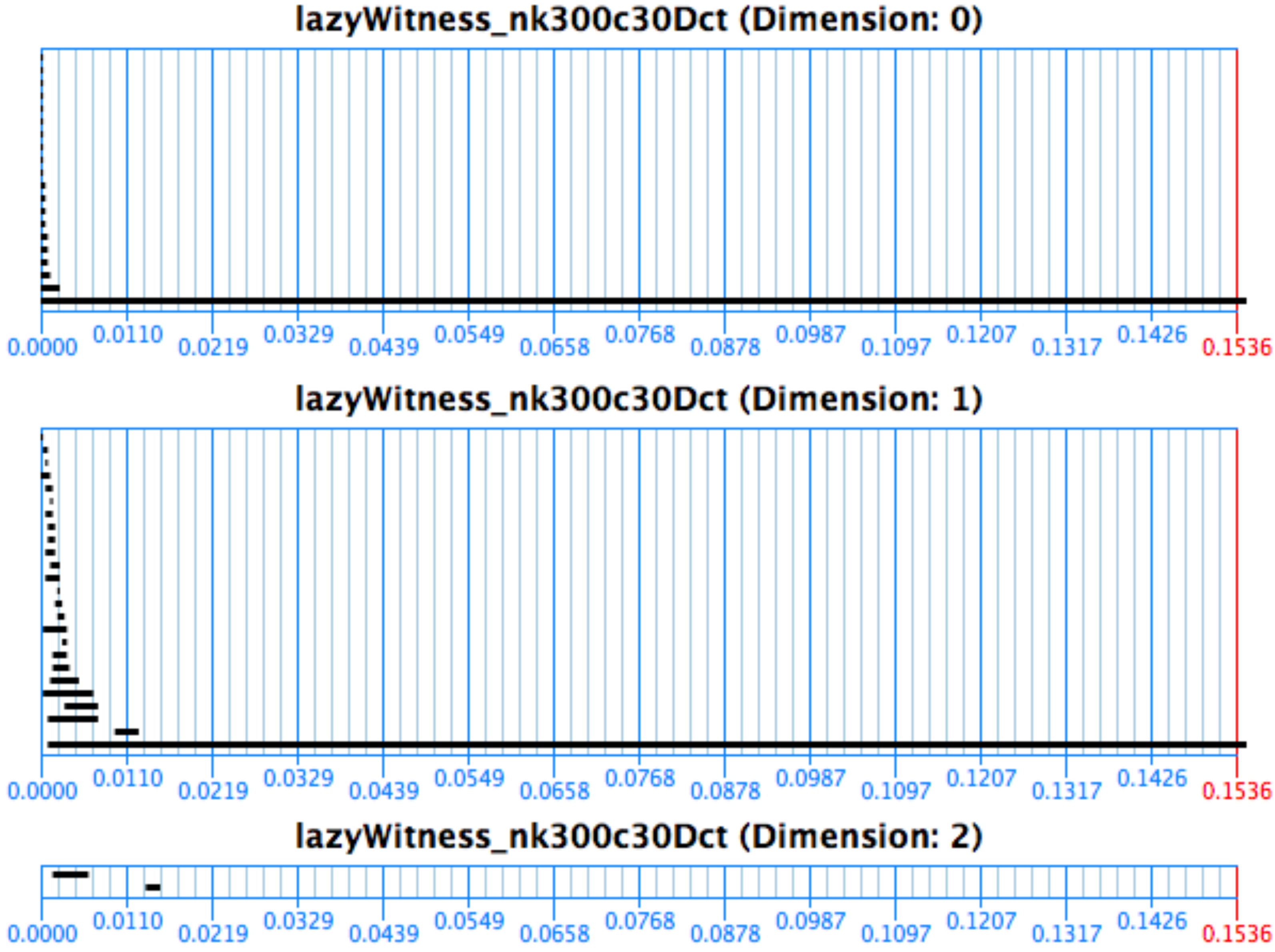
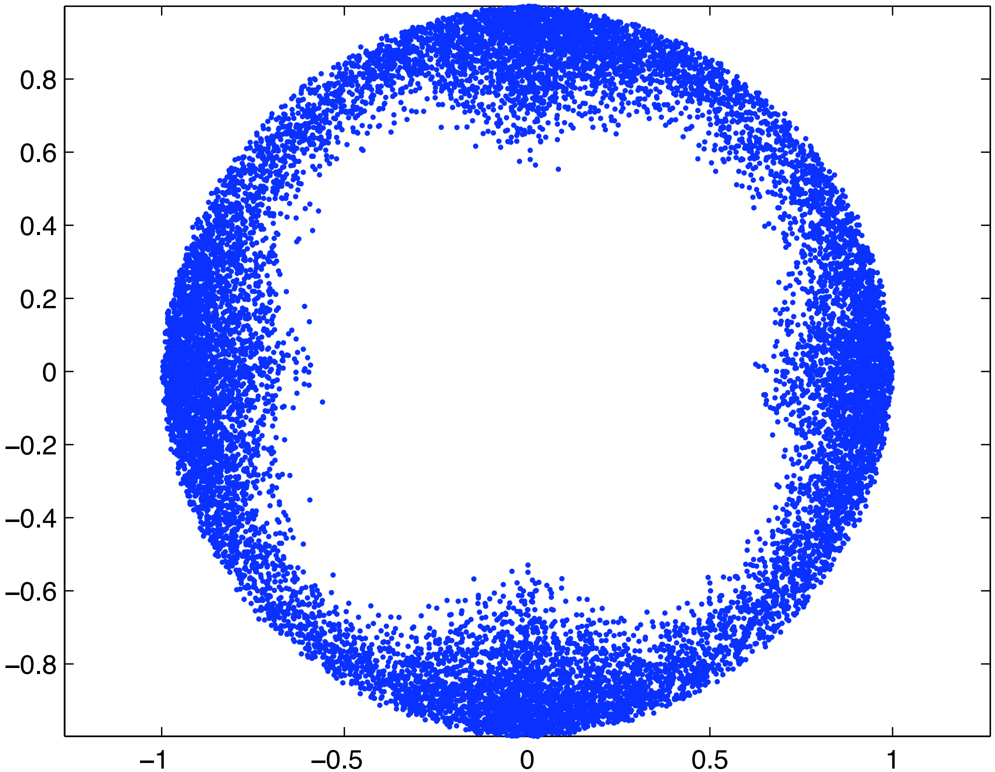
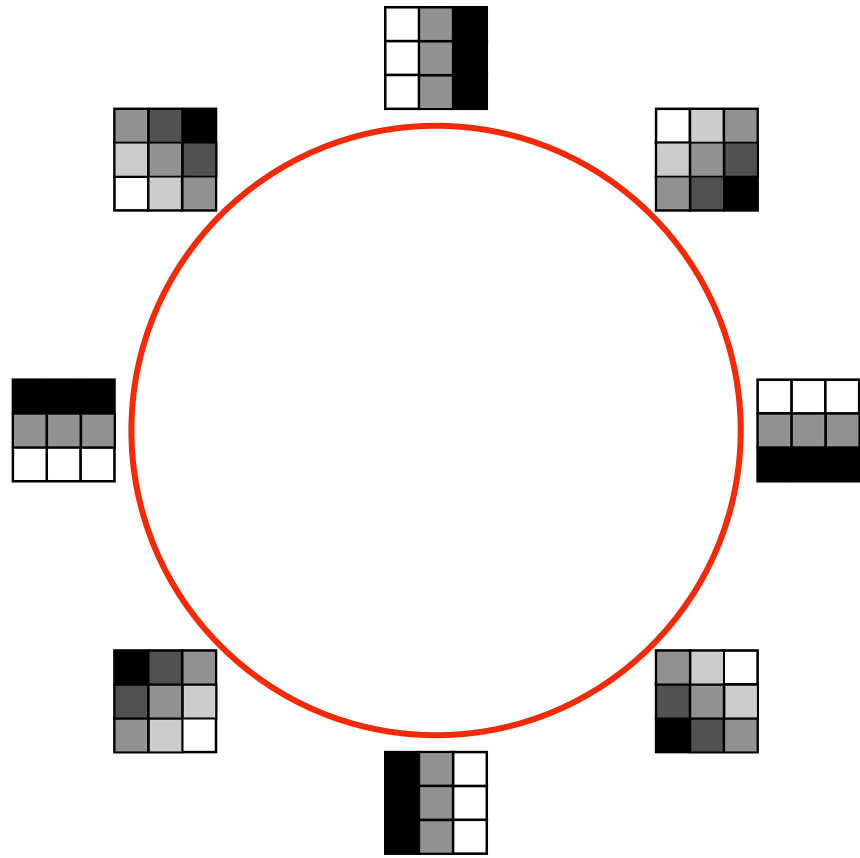


Image patch example

1st densest group of patches





Interpretation: nature prefers linearity

Image patch example

2nd densest group of patches

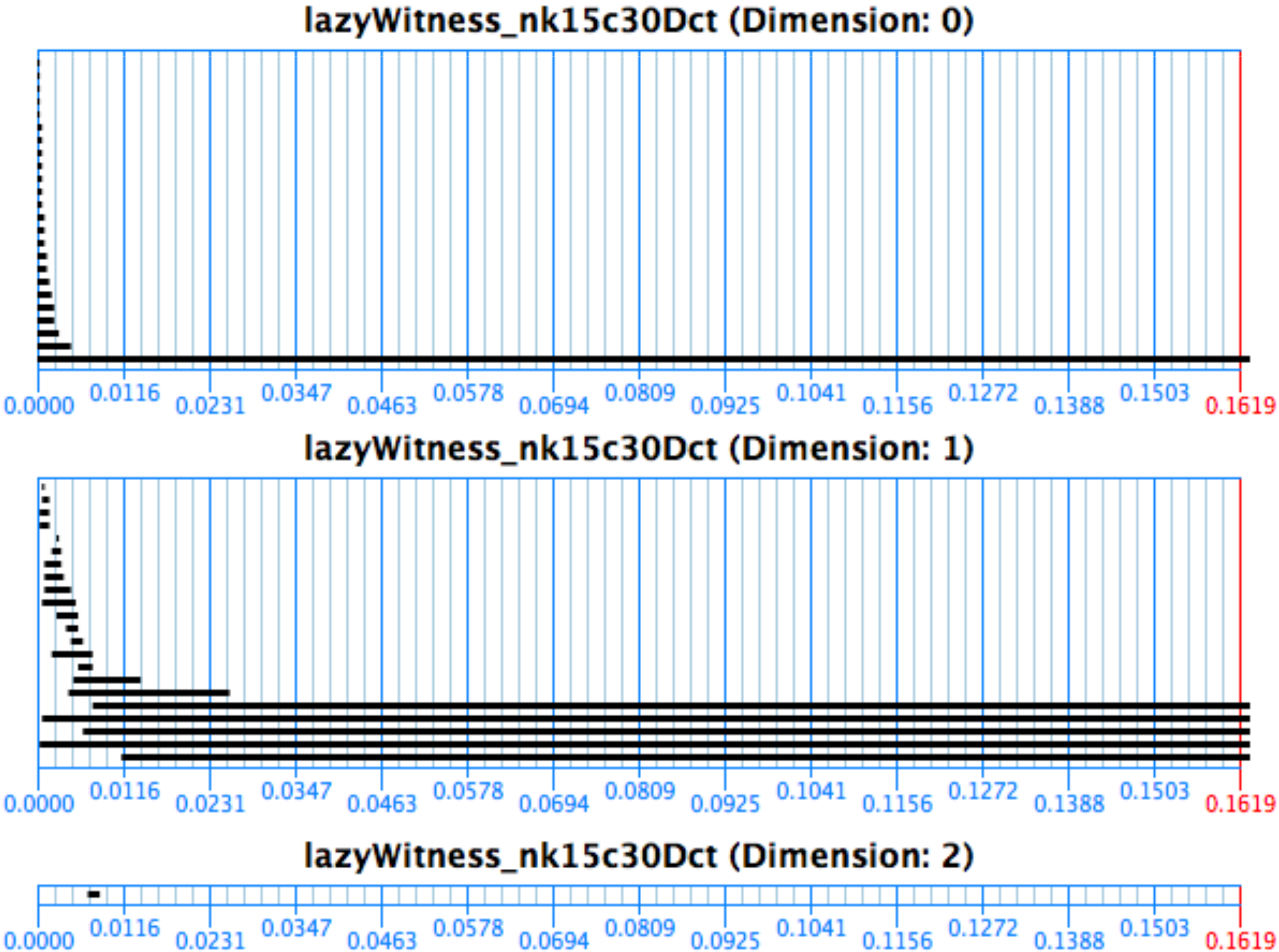
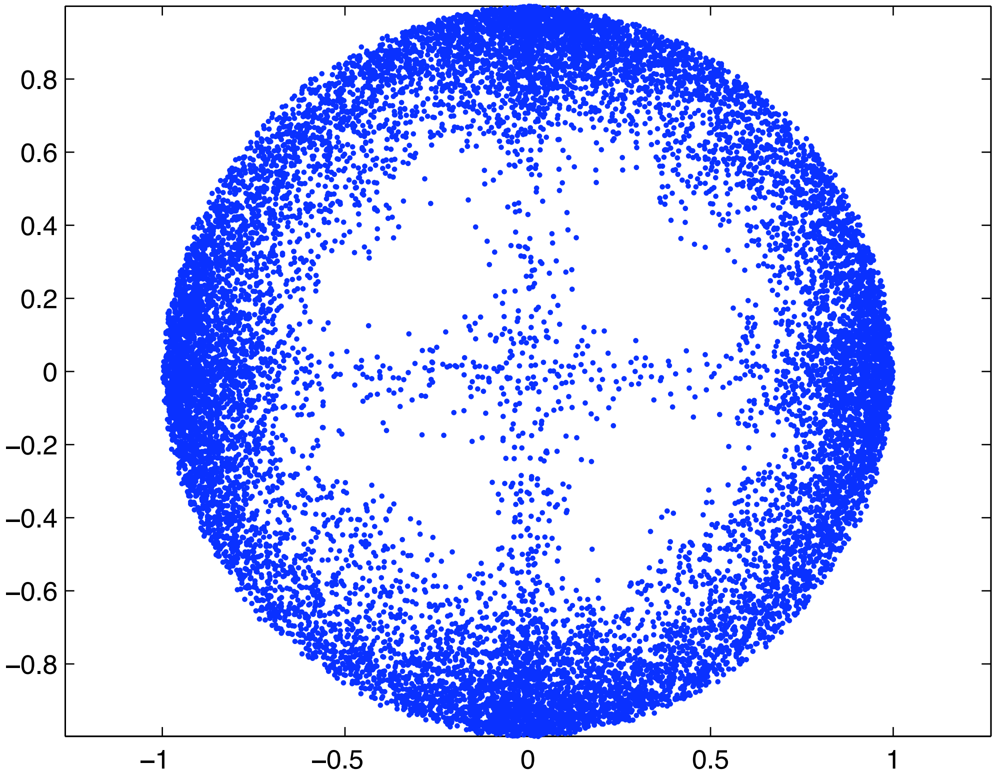
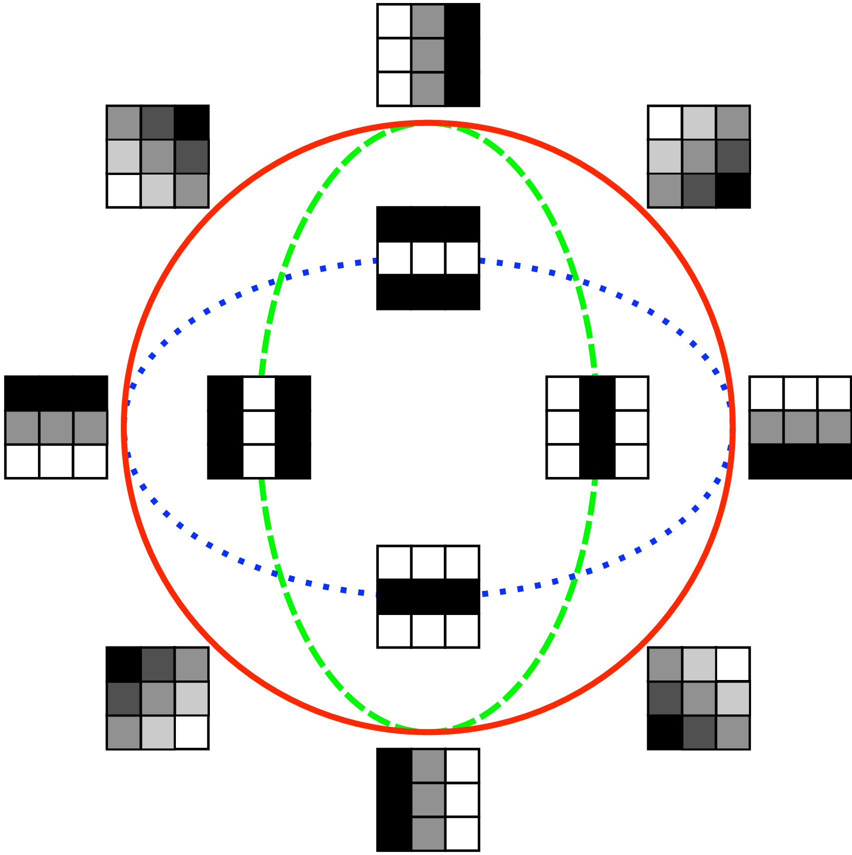


Image patch example

2nd densest group of patches





Interpretation: nature prefers horizontal and vertical directions

Image patch example

3rd densest group of patches

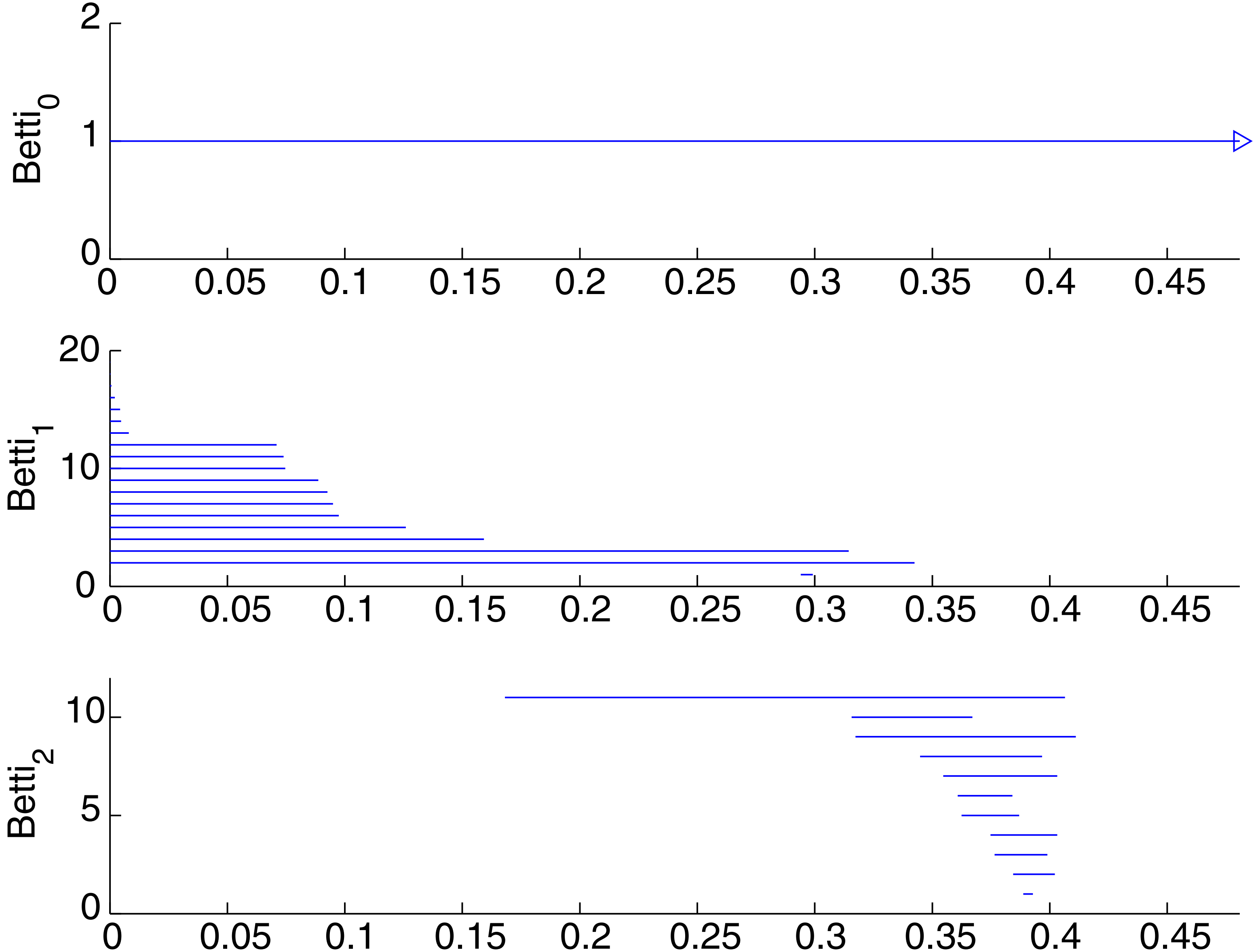


Image patch example

3rd densest group of patches

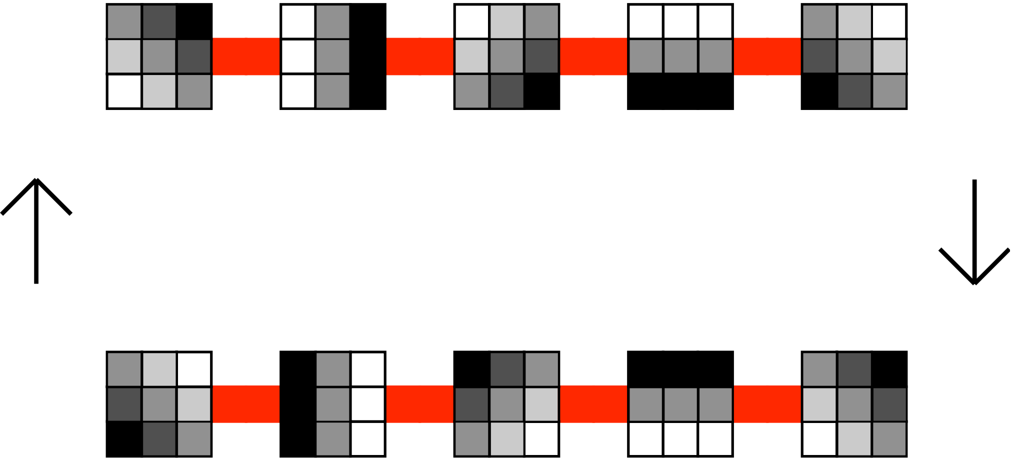


Image patch example

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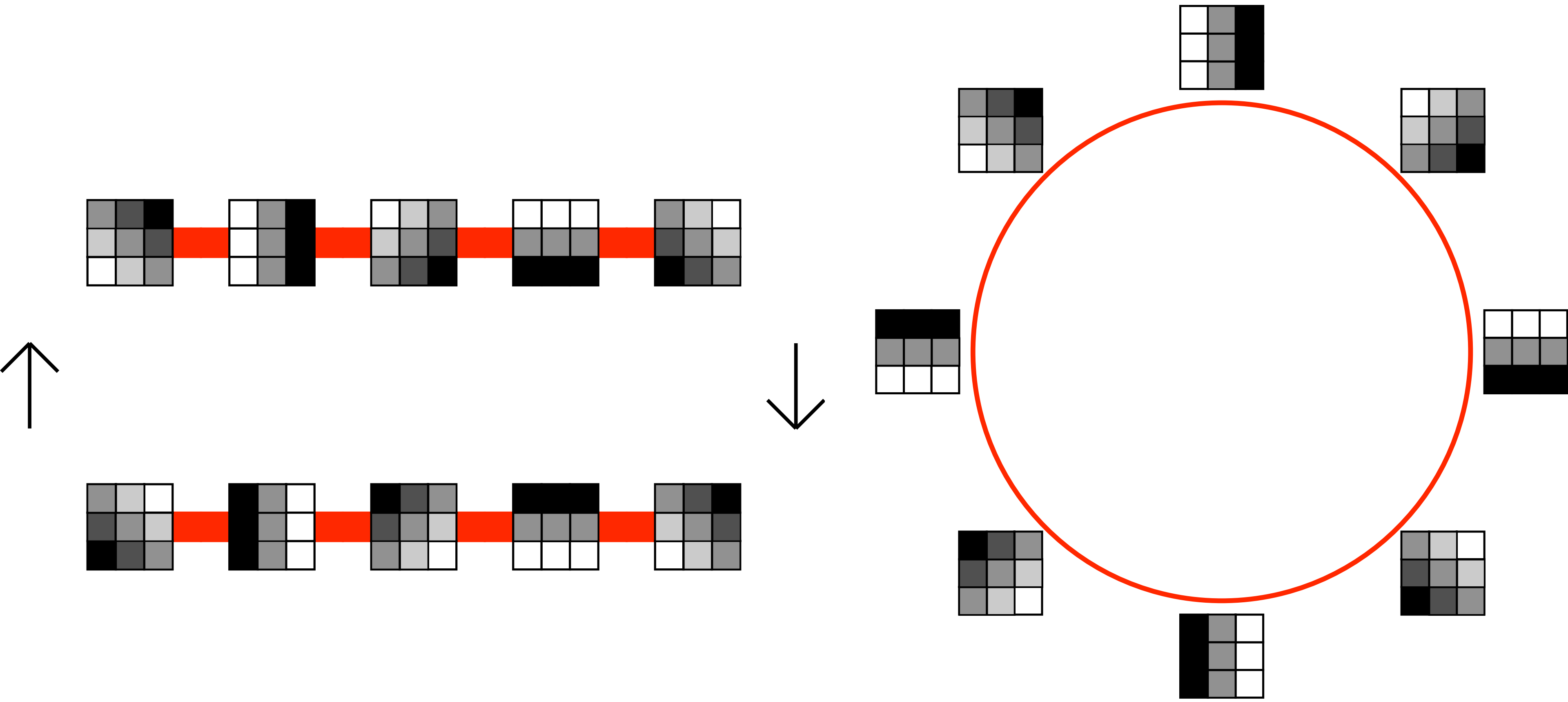

Image patch example

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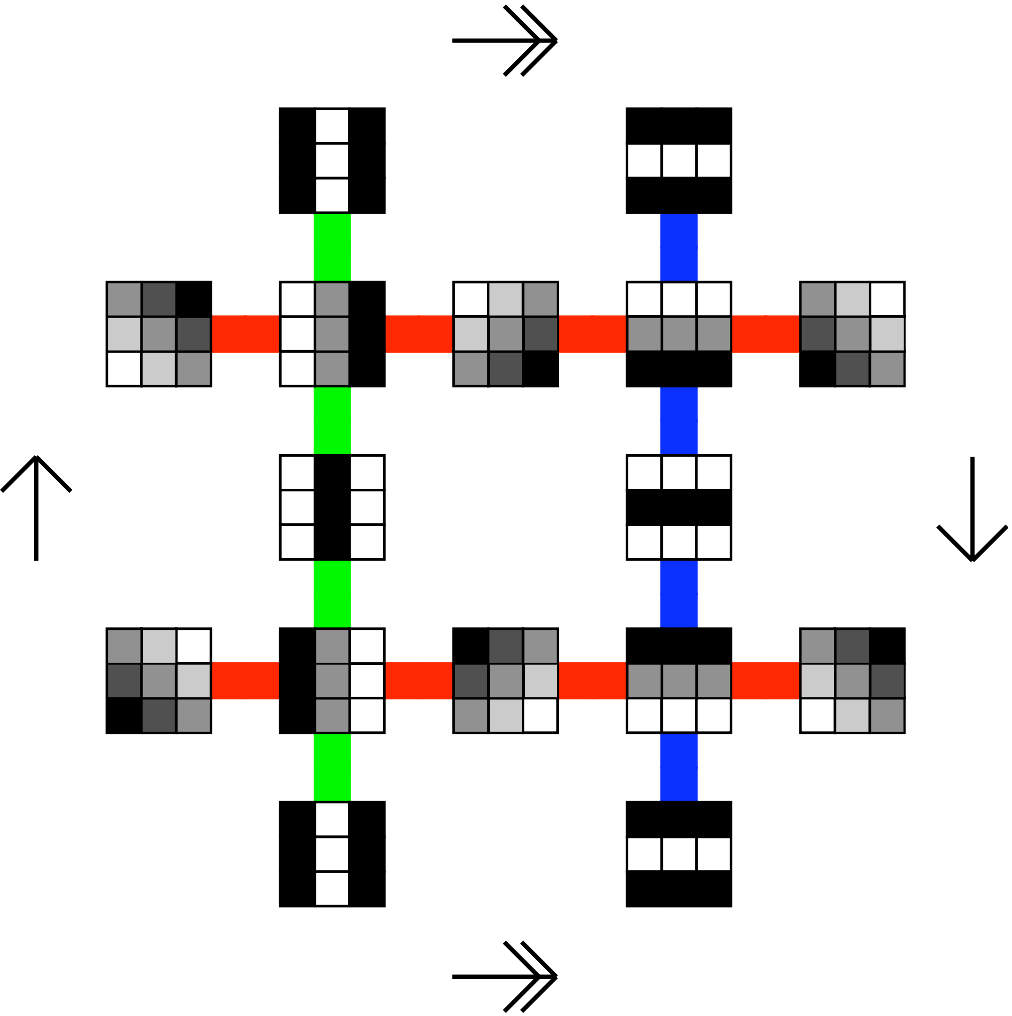

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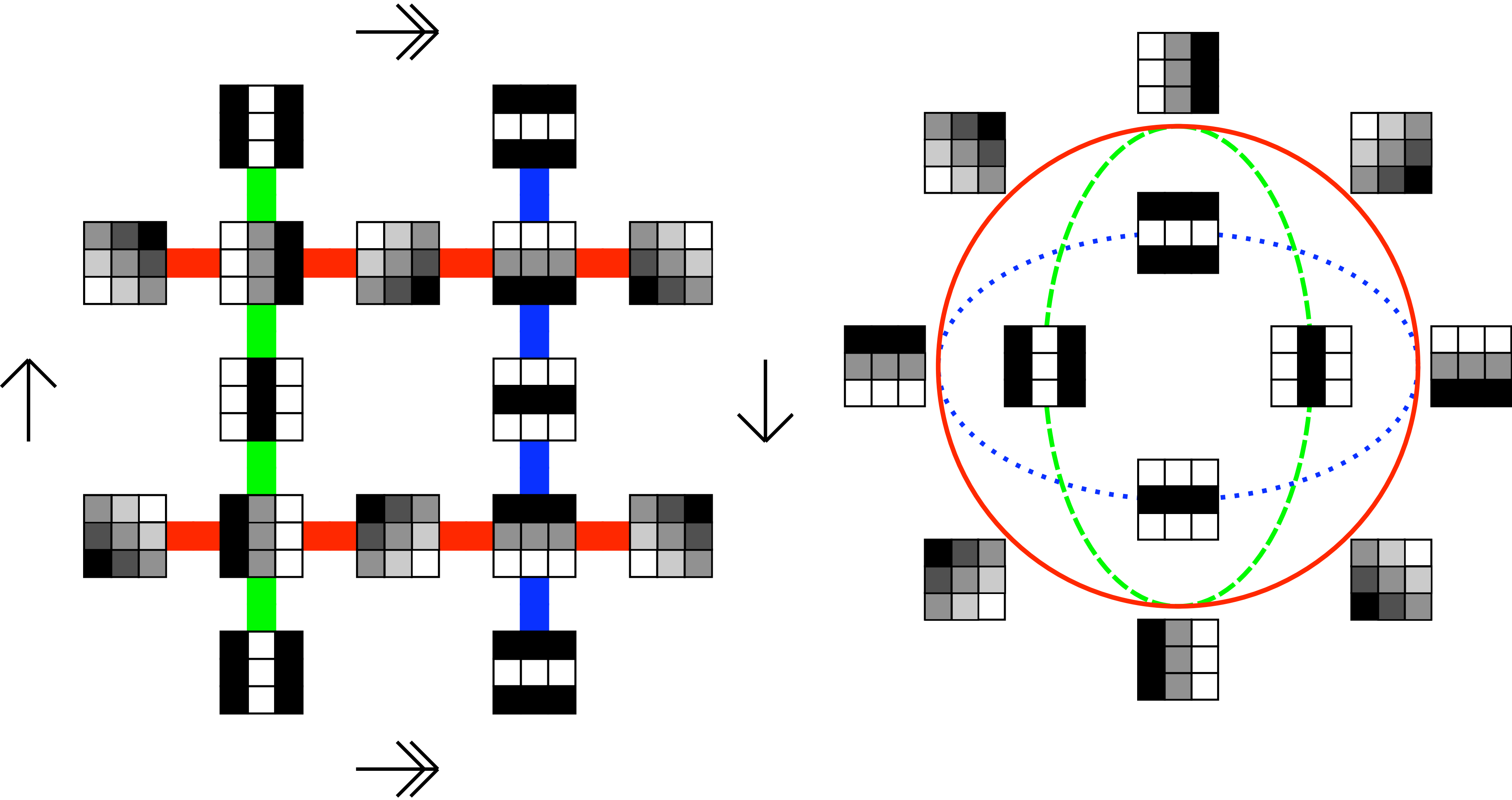

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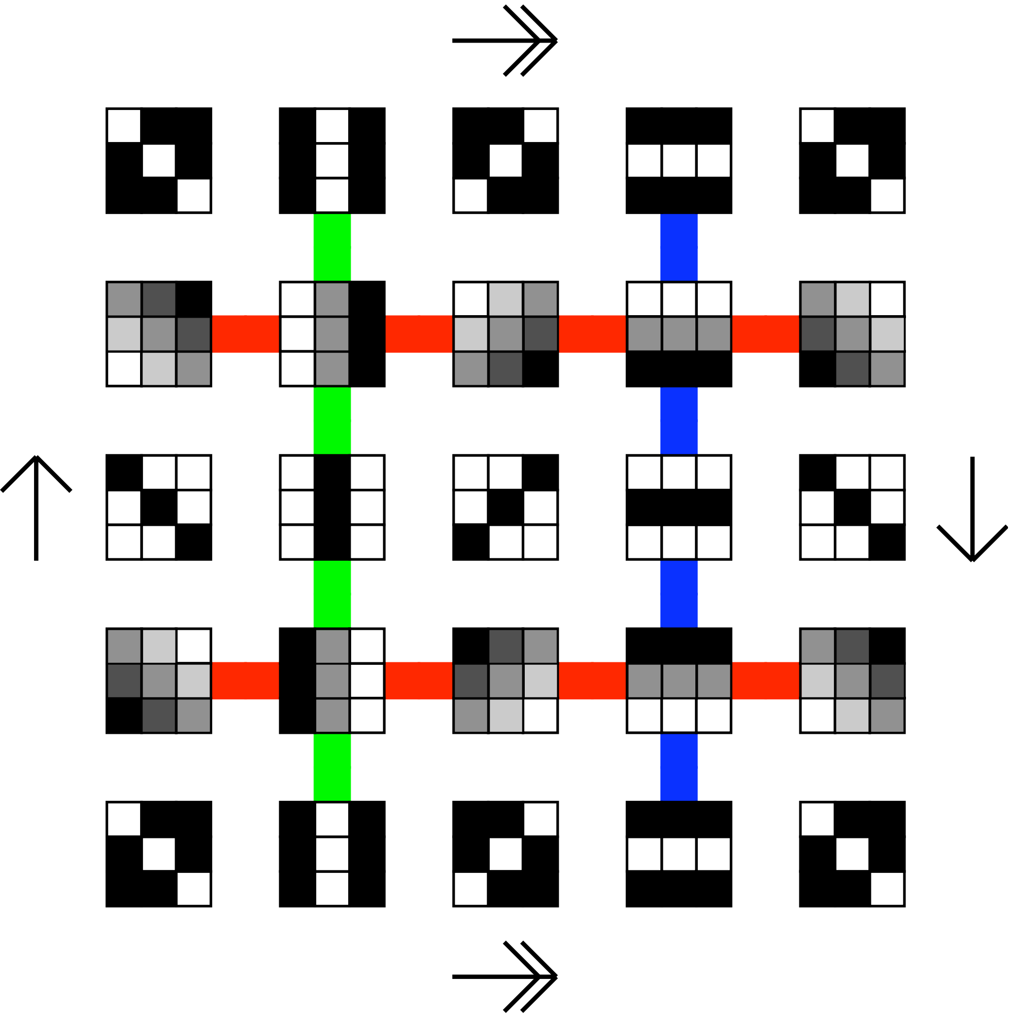

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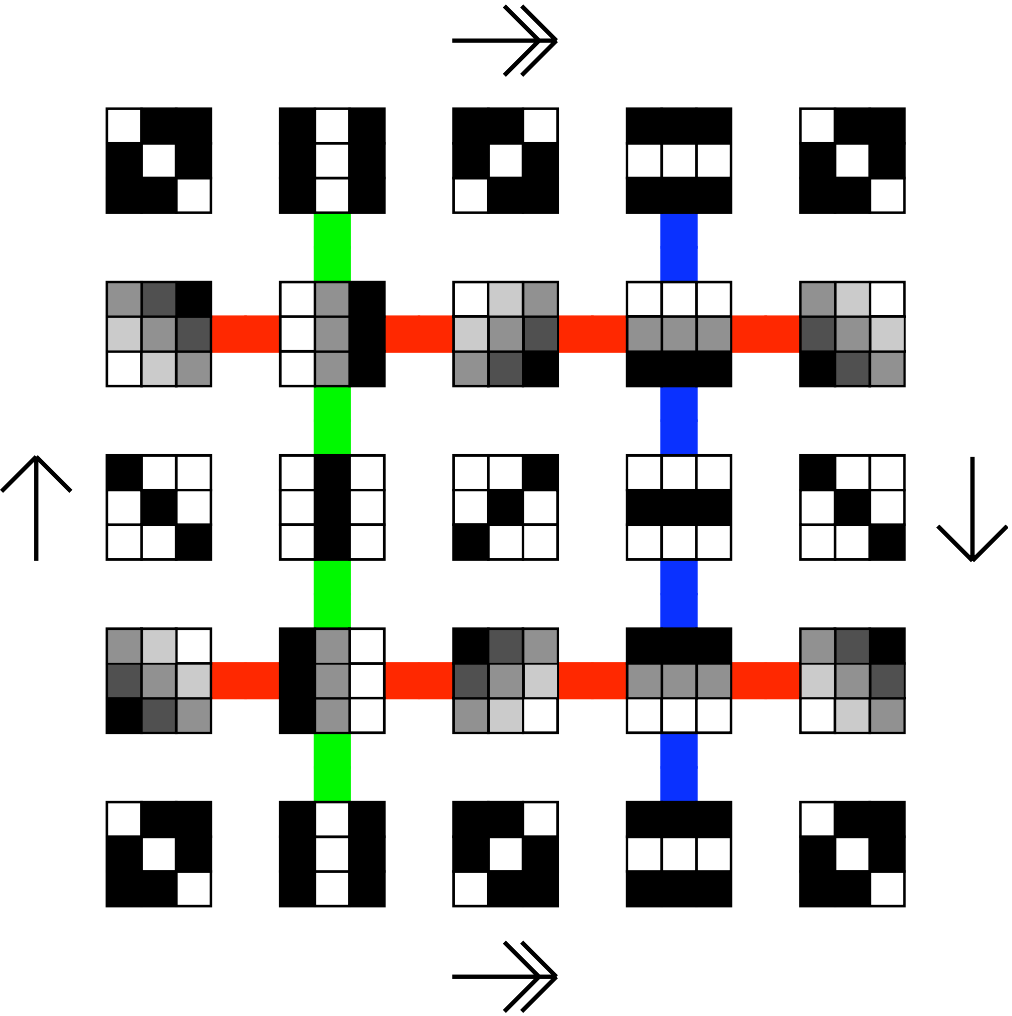

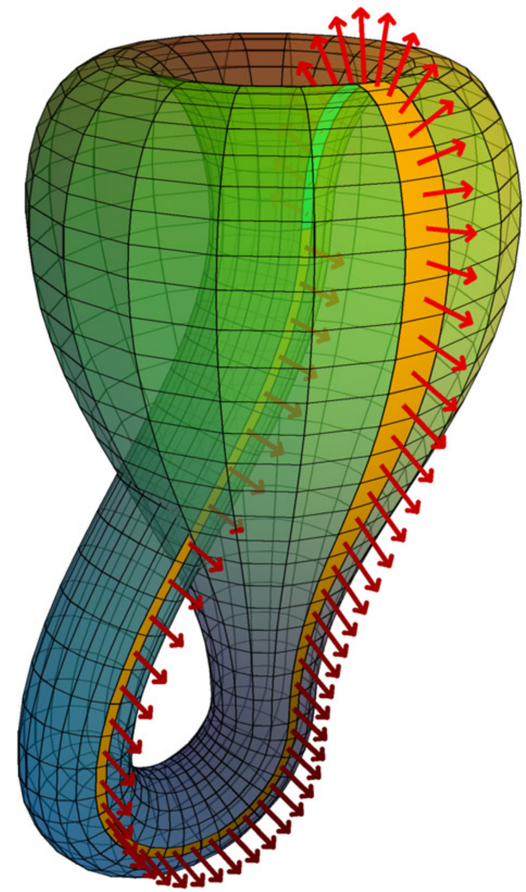

https://plus.maths.org/content/imaging-maths-inside-klein-bottle

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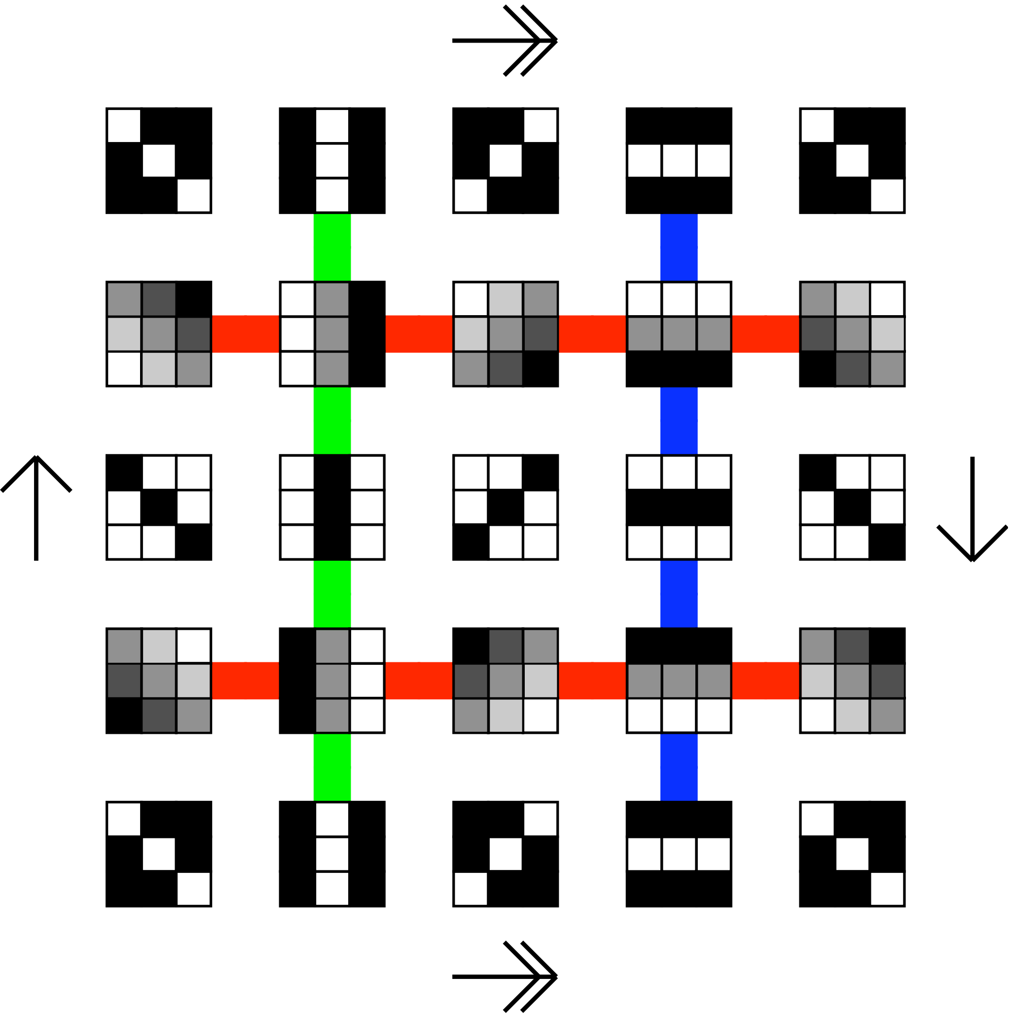
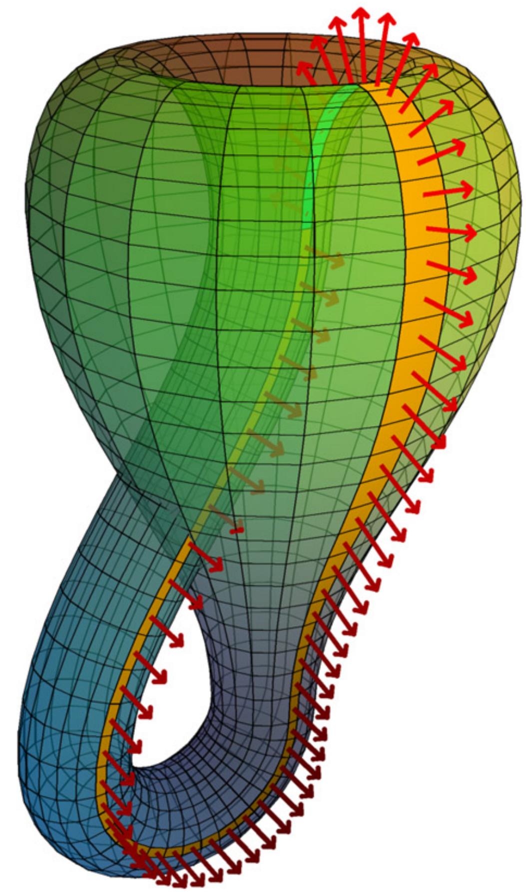


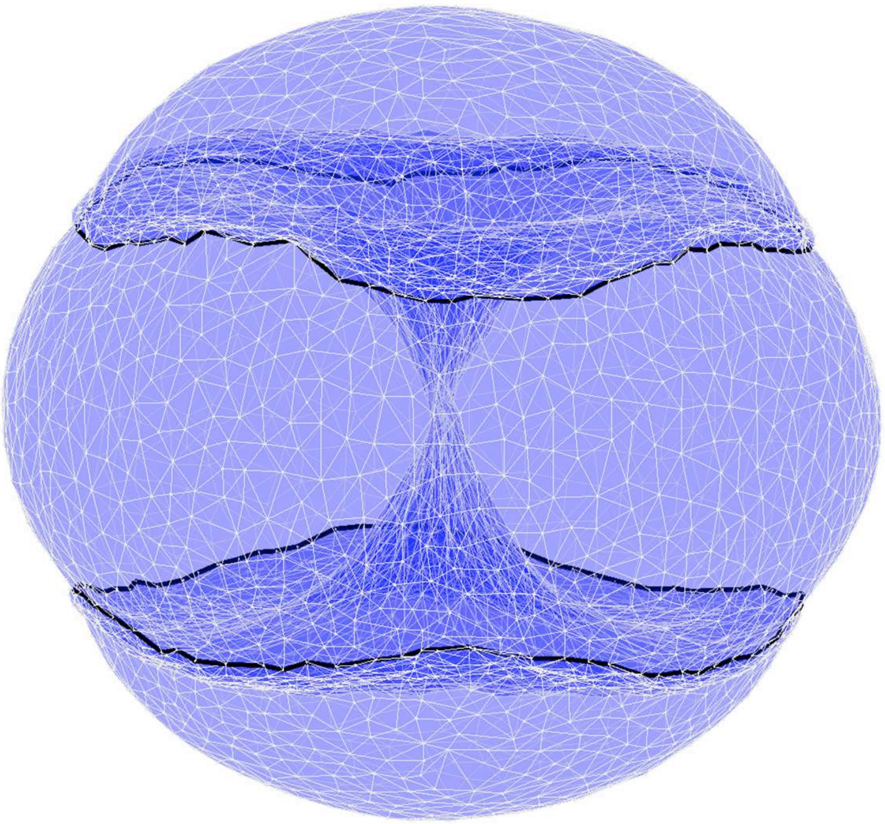


Interpretation: nature prefers linear and quadratic patches at all angles

Datasets have shapes

Example: Cyclo-Octane (C_8H_{16}) data

1,031,644 points in 72-dimensional space

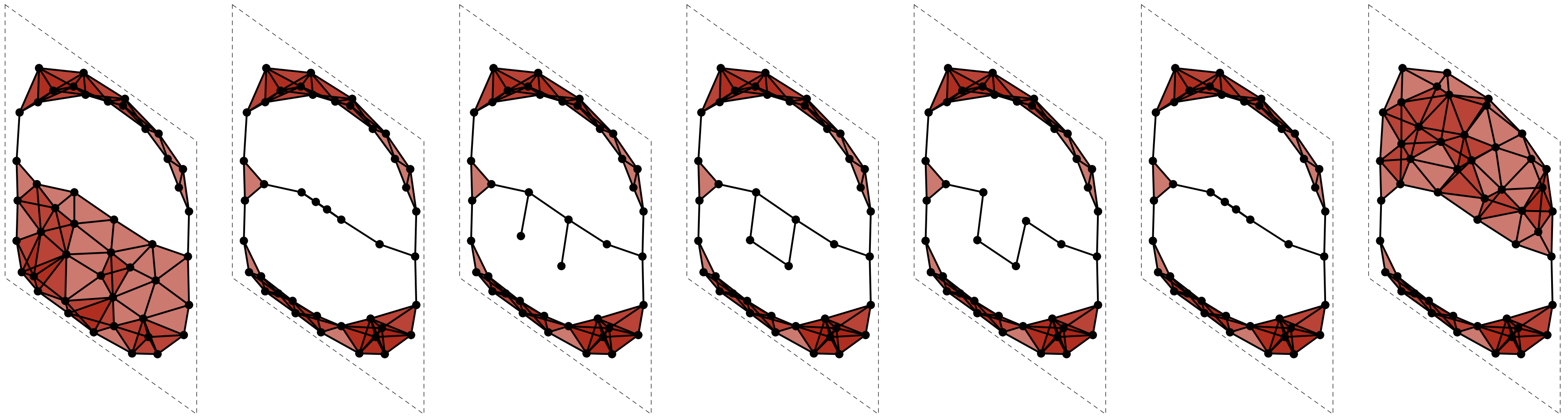


Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data by Shawn Martin and Jean-Paul Watson, 2010.

References

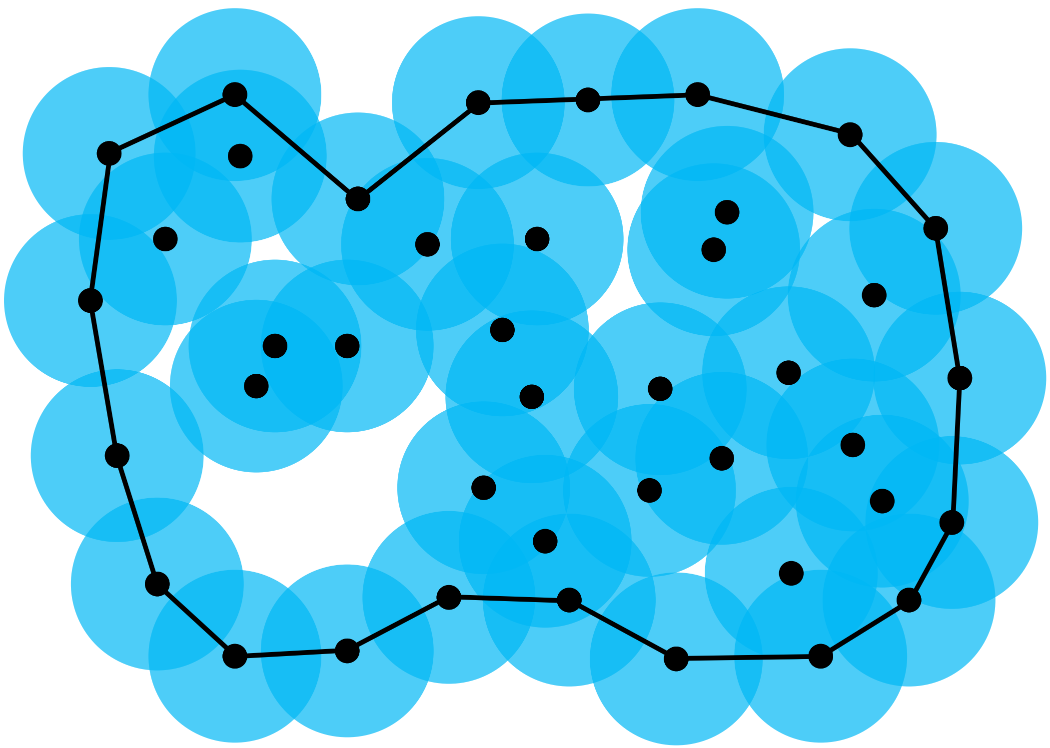
- *An Attempt to Define the Nature of Chemical Diabetes Using a Multidimensional Analysis* by G. M. Reaven and R. G. Miller, 1979.
- *Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data* by Shawn Martin and Jean-Paul Watson, 2010.
- *On the Local Behavior of Spaces of Natural Images* by Gunnar Carlsson, Tigran Ishkhanov, Vin de Silva, and Afra Zomorodian, 2008.

Evasion paths in mobile sensor networks



Coverage problem

- Sensors in a domain $\mathcal{D} \subset \mathbb{R}^n$.



Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology by V. de Silva and R. Ghrist

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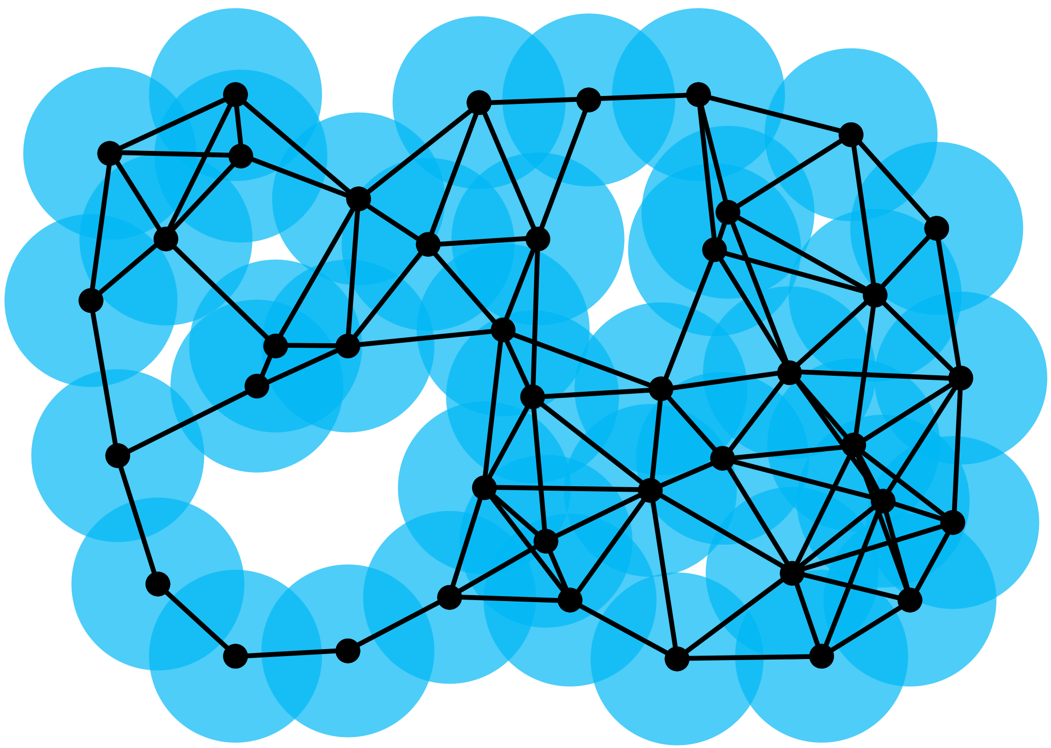
- Sensors in a domain $\mathcal{D} \subset \mathbb{R}^n$.
- Measure only local connectivity.



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- Sensors in a domain $\mathcal{D} \subset \mathbb{R}^n$.
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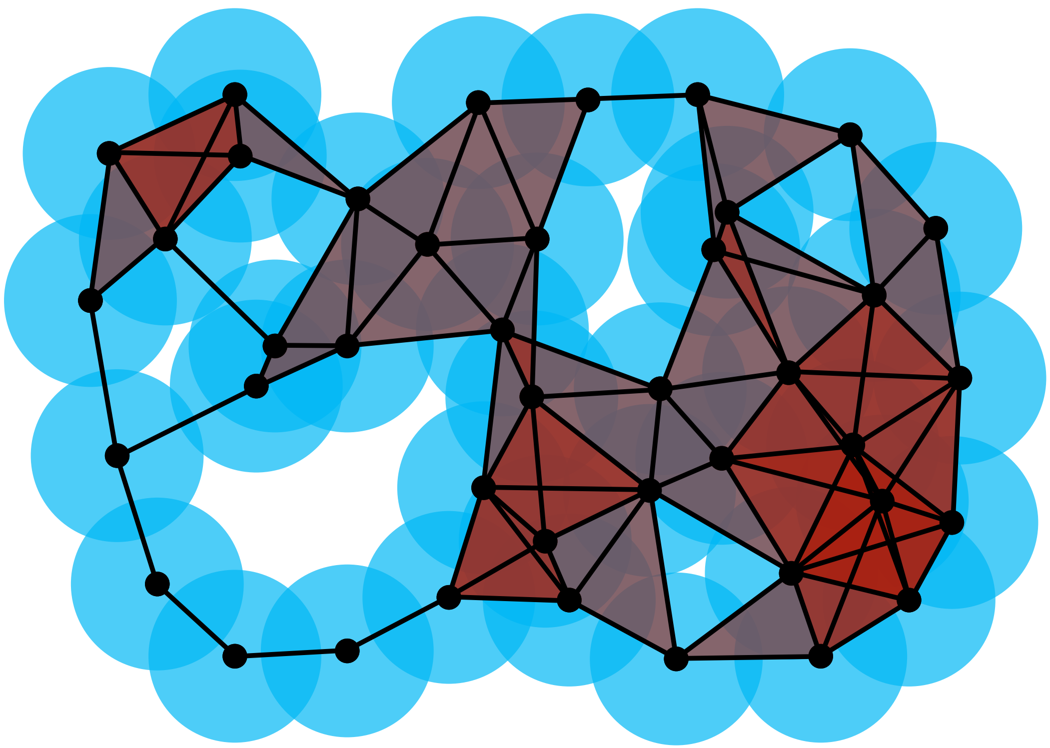

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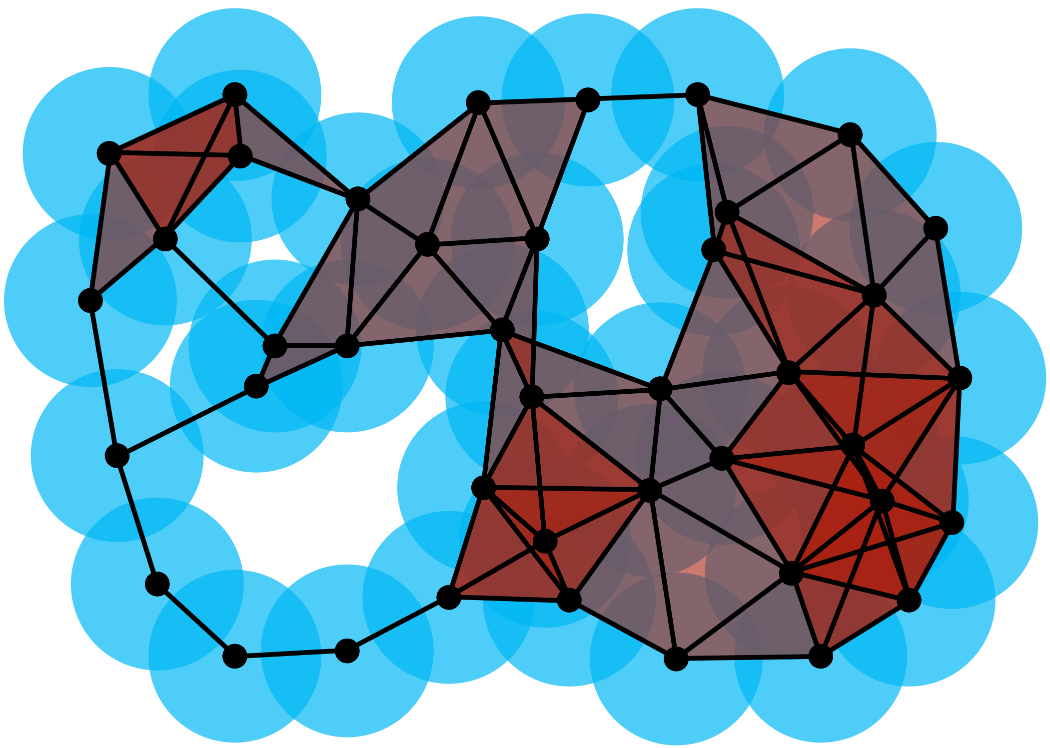

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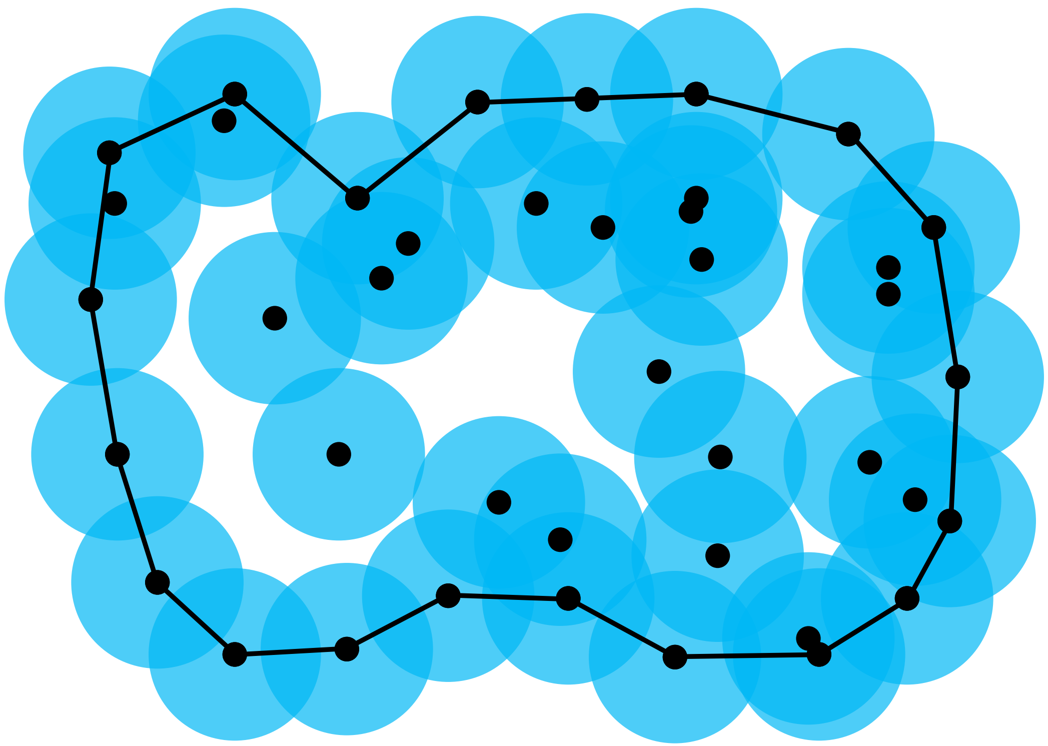

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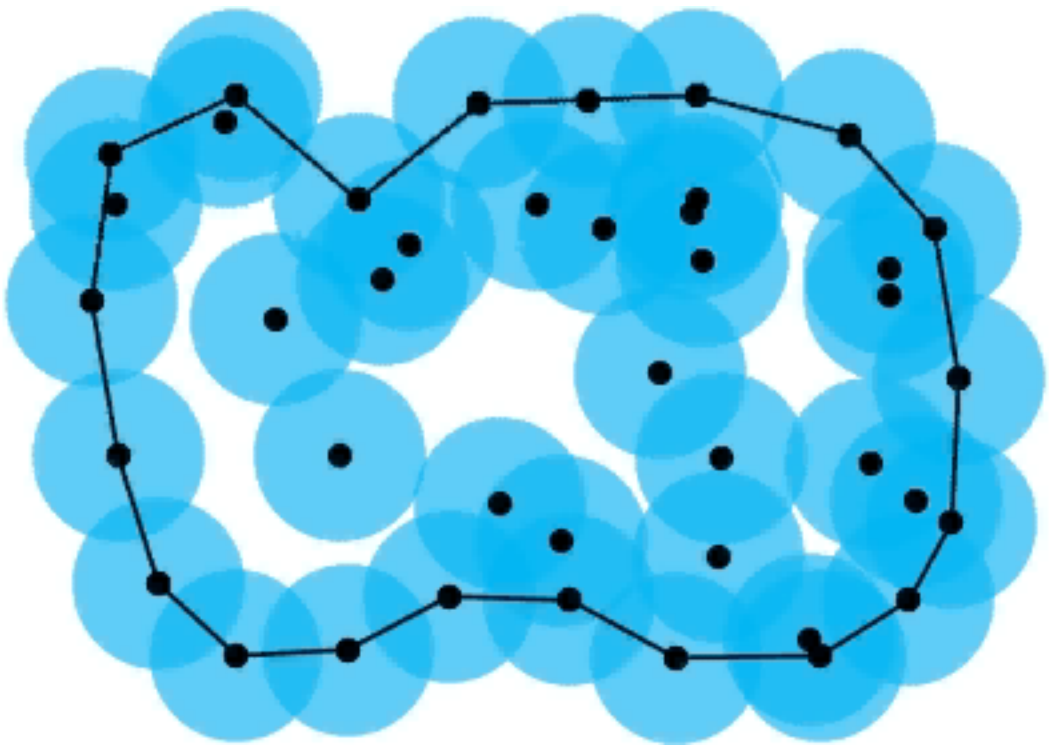

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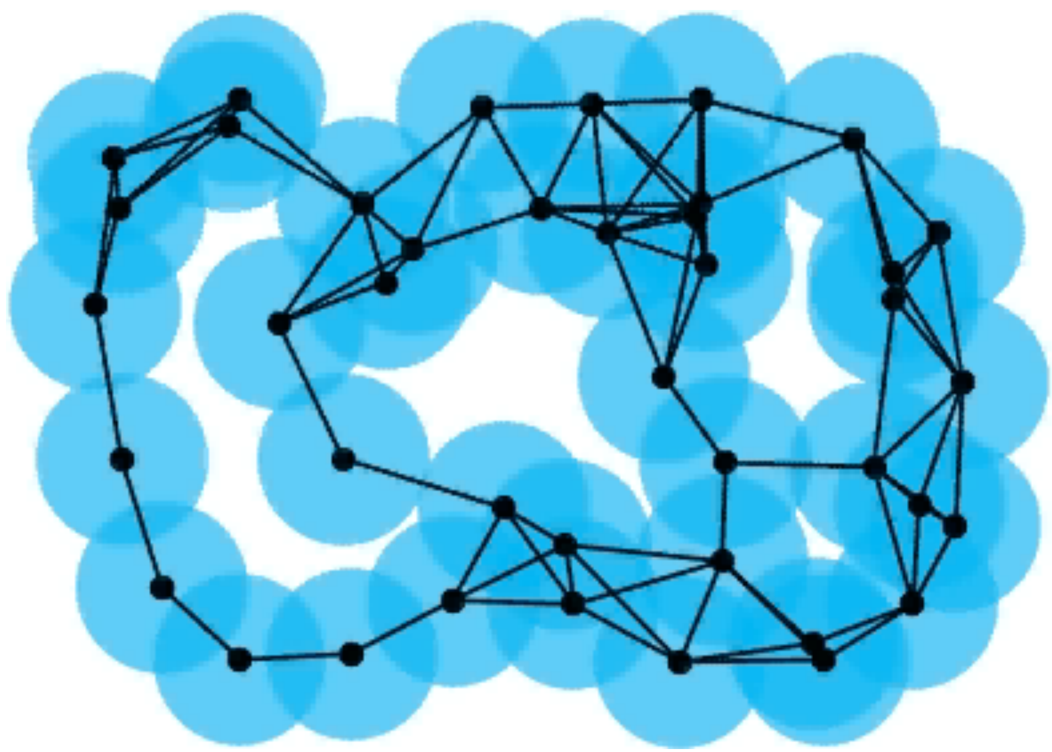

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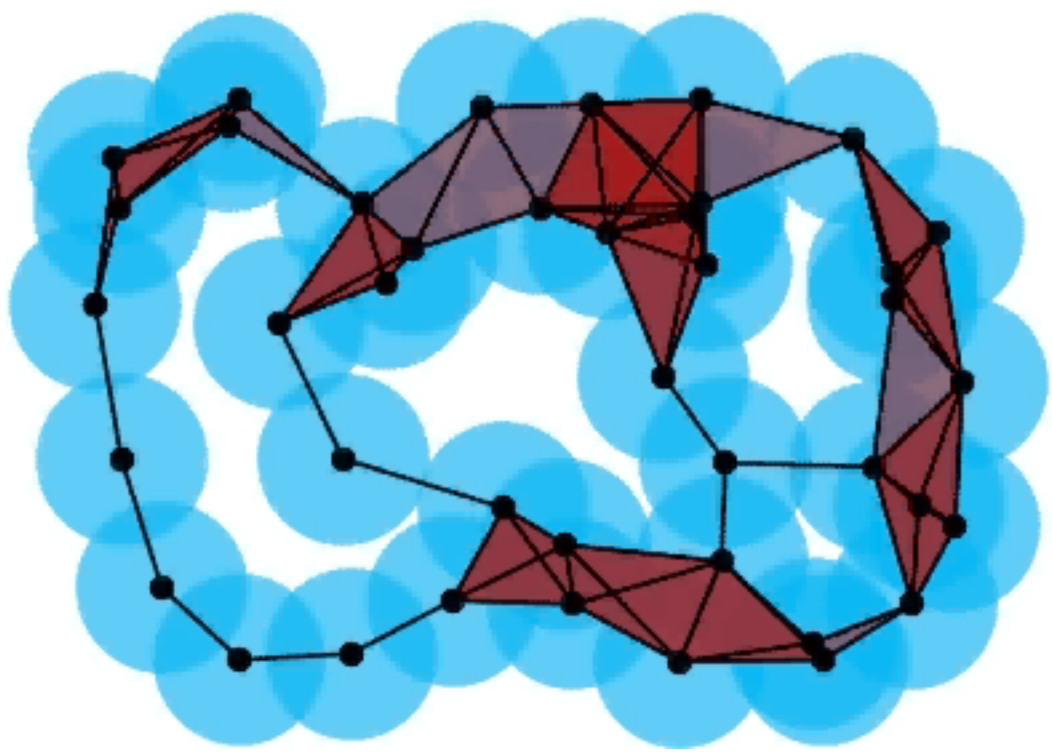

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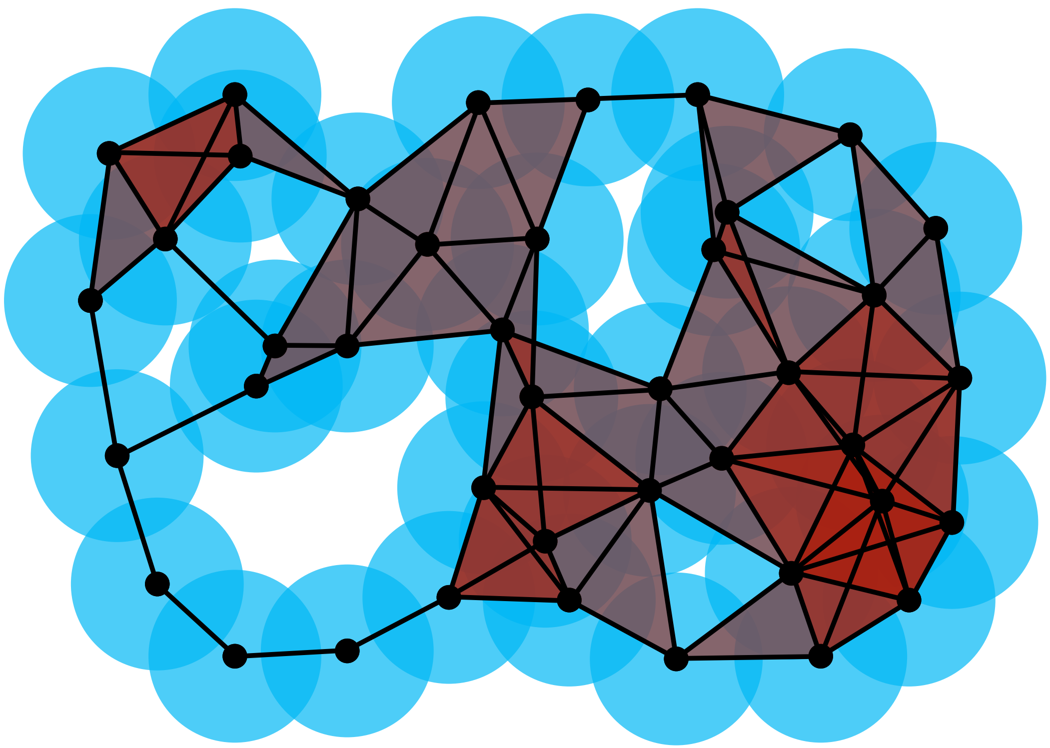

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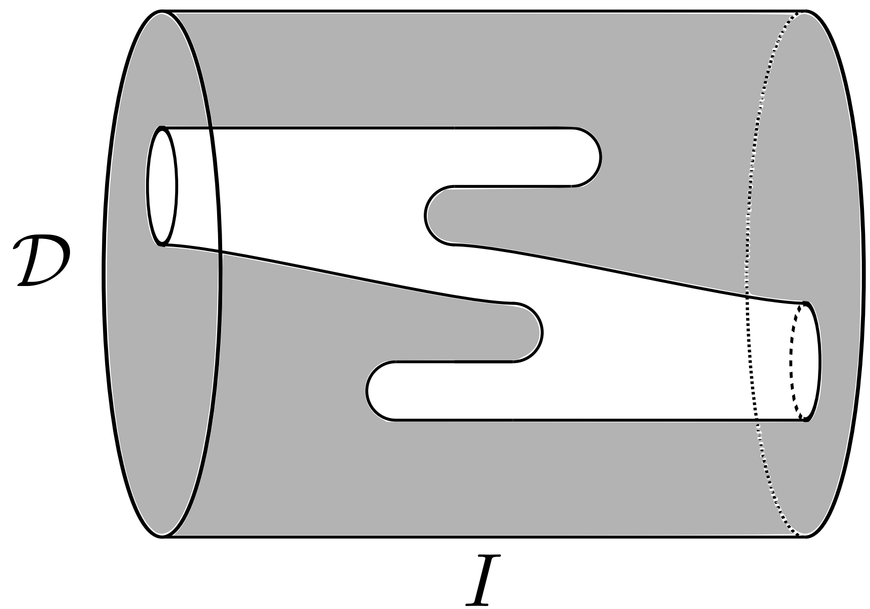
- Let $X \subset \mathcal{D} \times I$ be the covered region.



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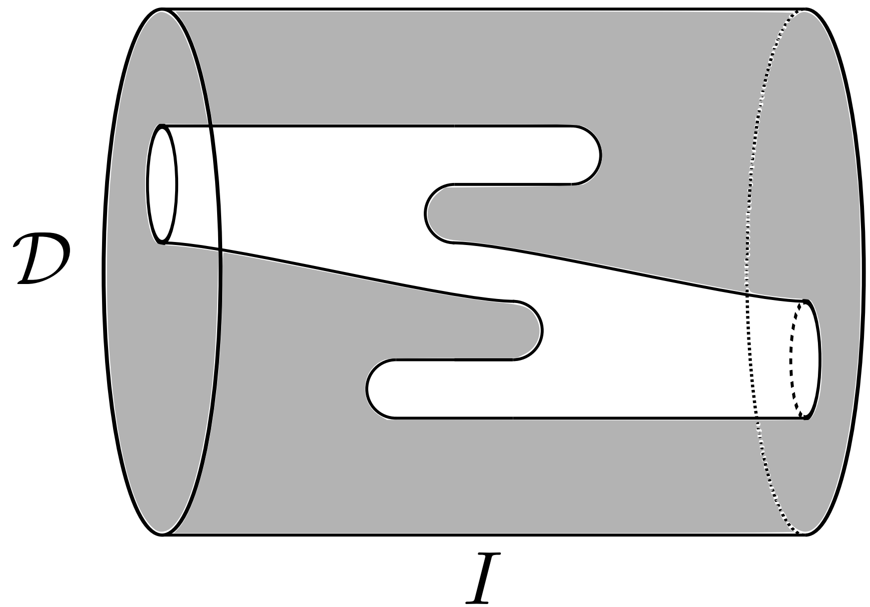

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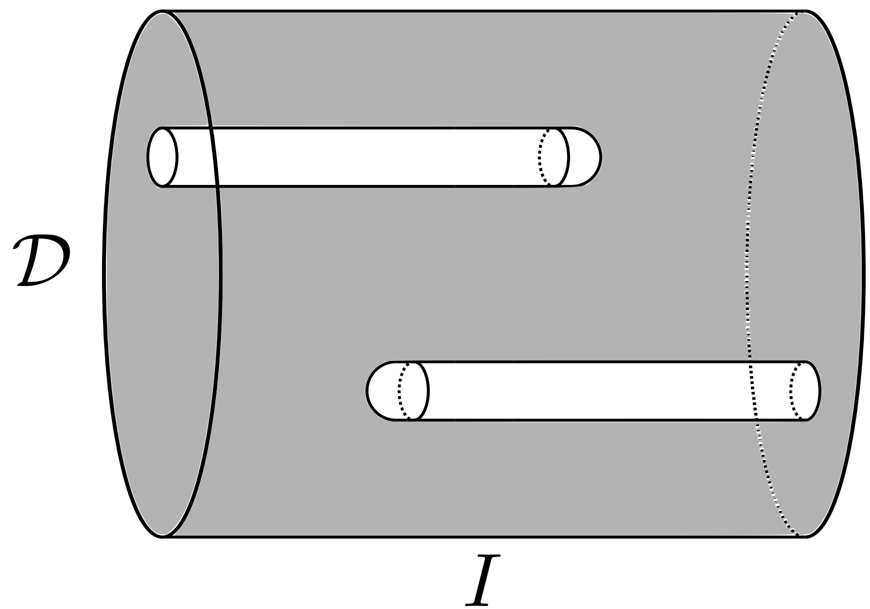

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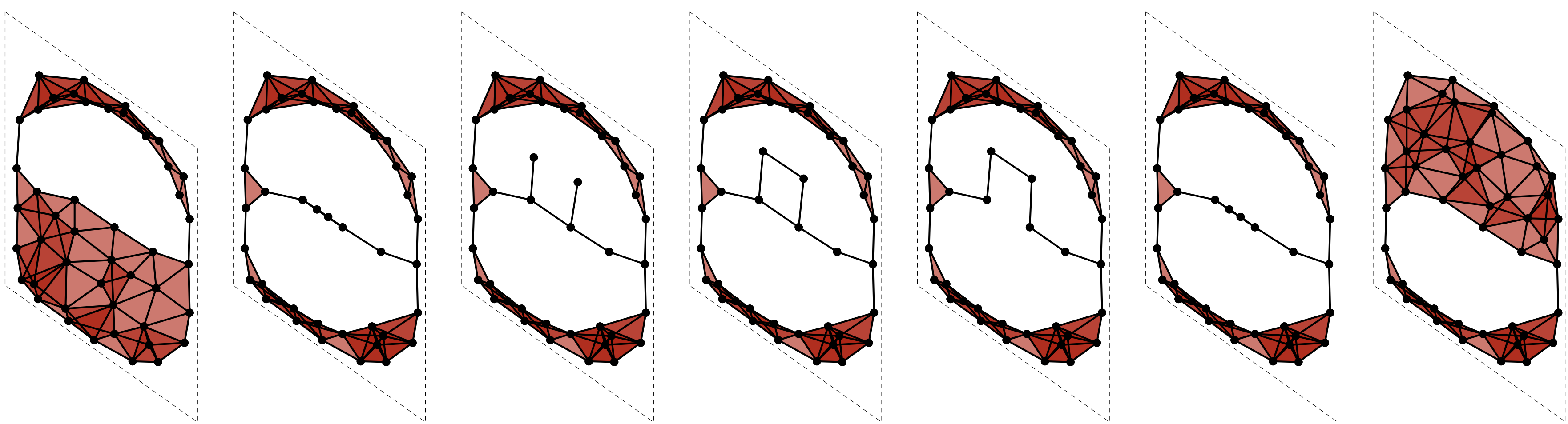
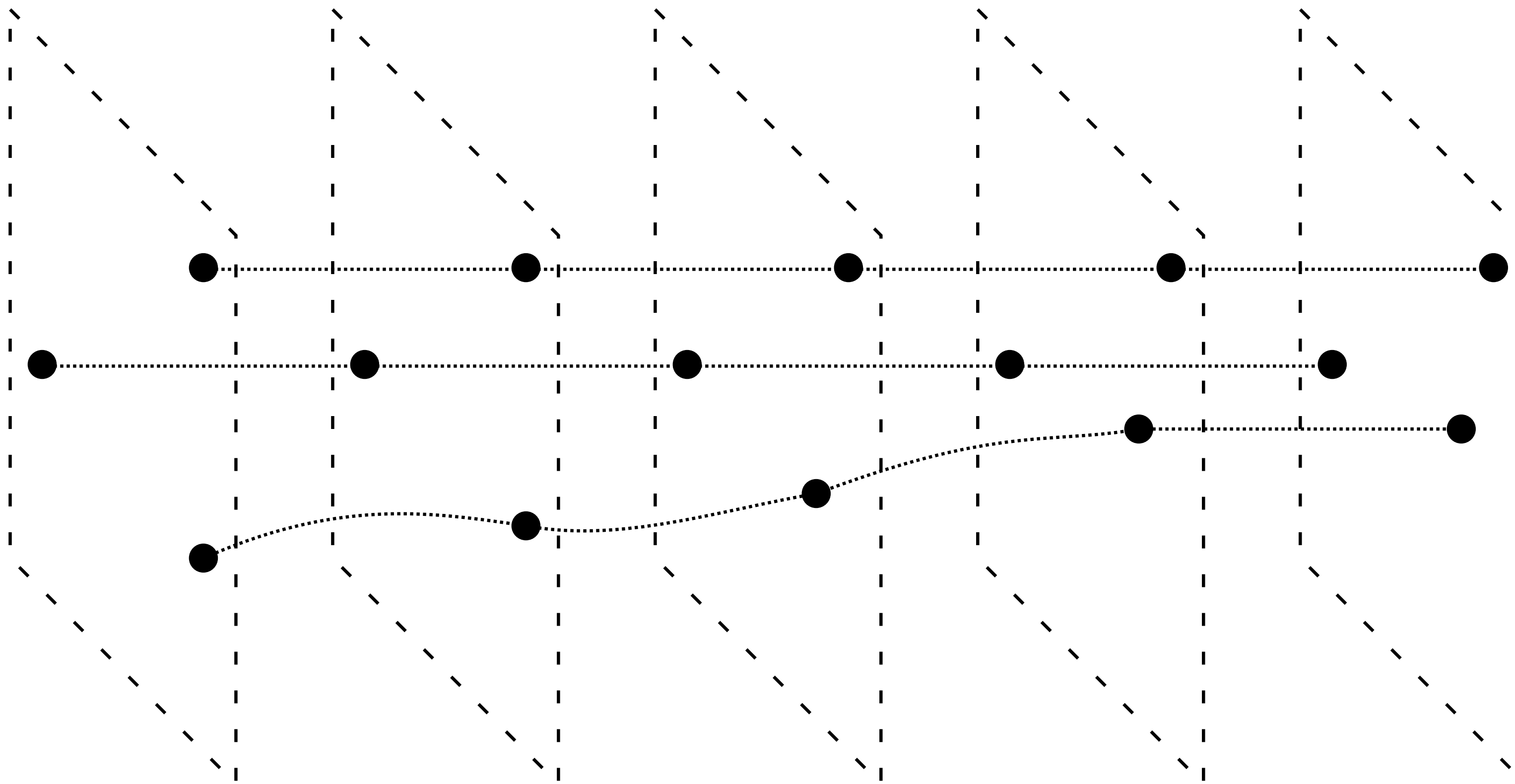


Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology by V. de Silva and R. Ghrist

Evasion problem

- Stacked complex $\mathcal{SC} \simeq X$ encodes all Čech complexes.



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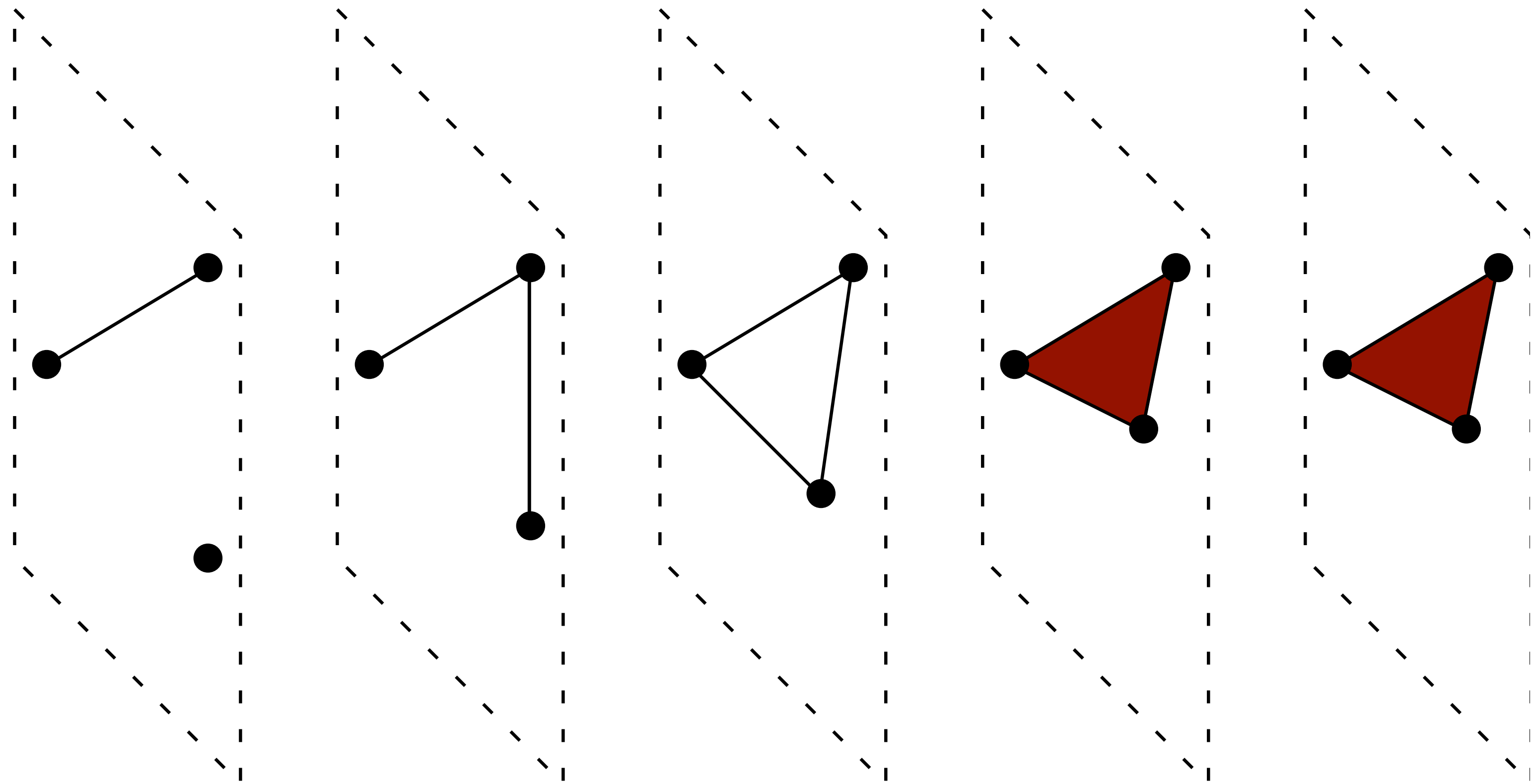

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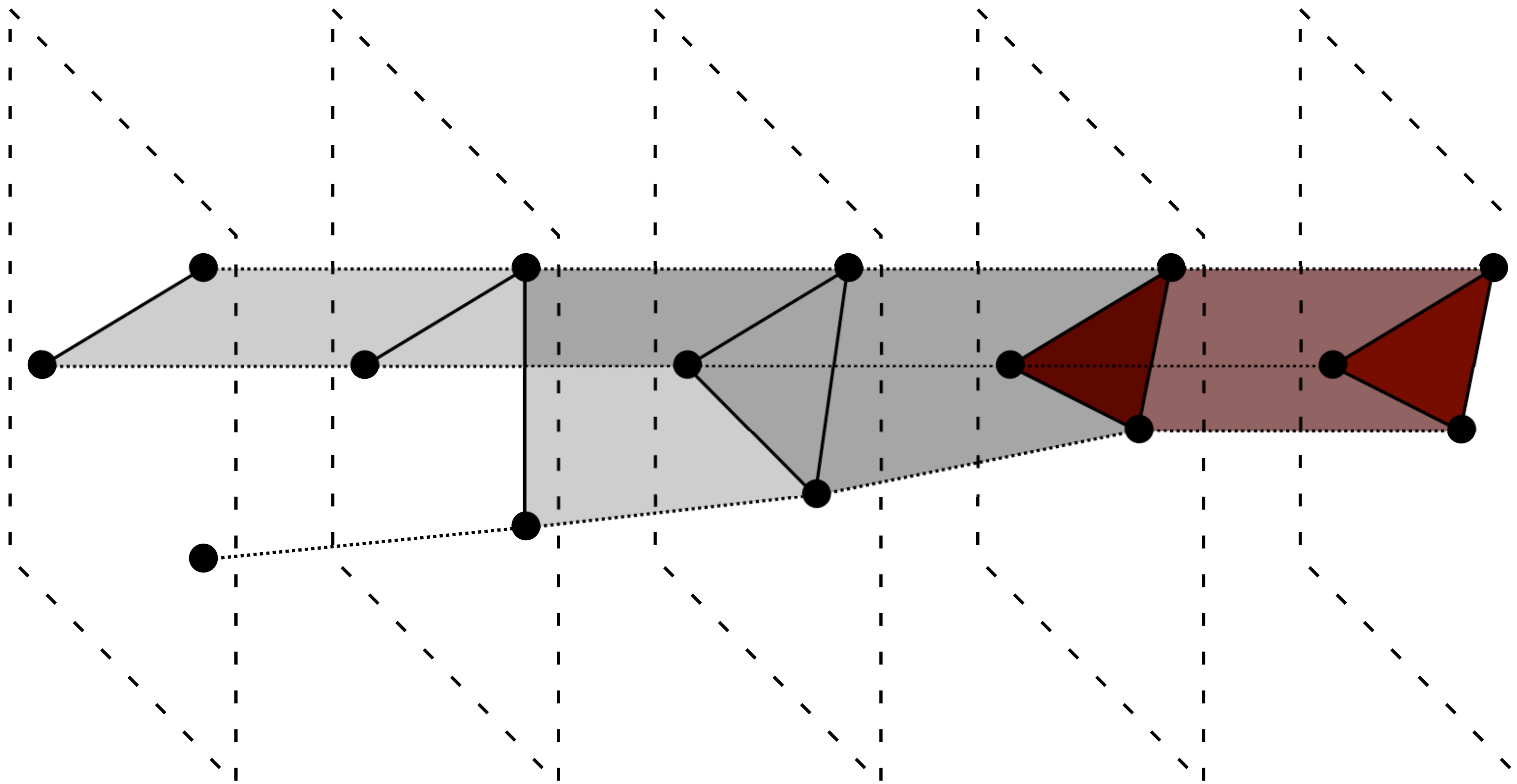

Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology by V. de Silva and R. Ghrist

Evasion problem

- <u>Theorem 7 (reformulated)</u>
  If there is an $\alpha \in H_n(\mathcal{SC}, \partial\mathcal{D} \times I)$ with
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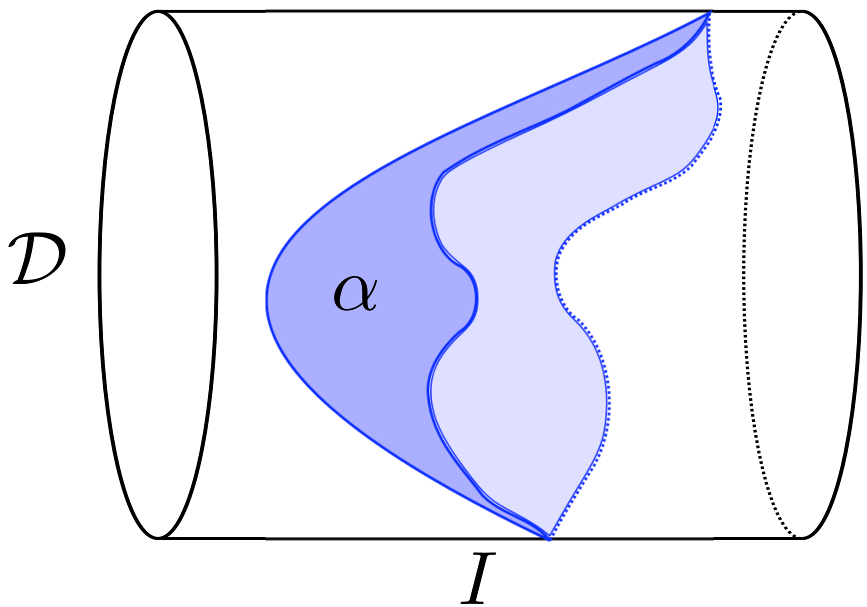

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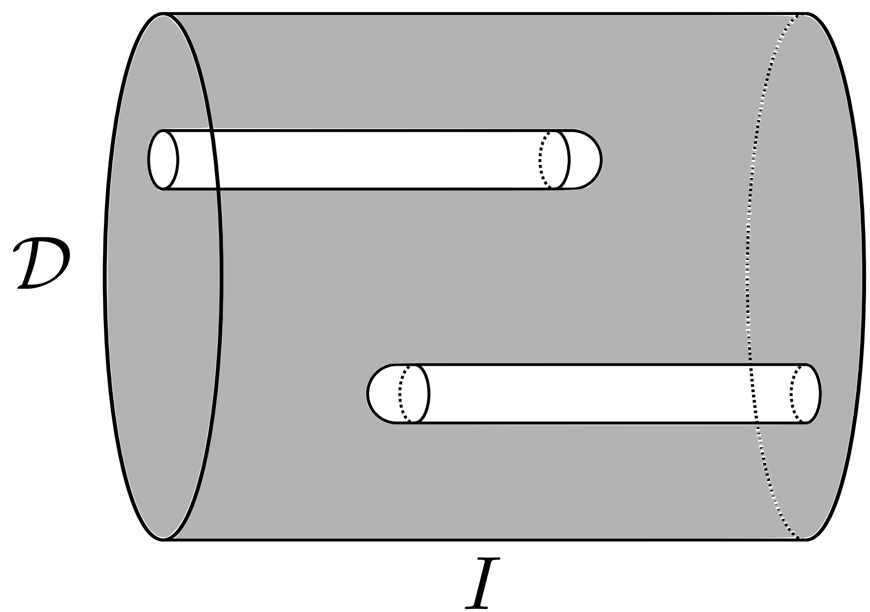

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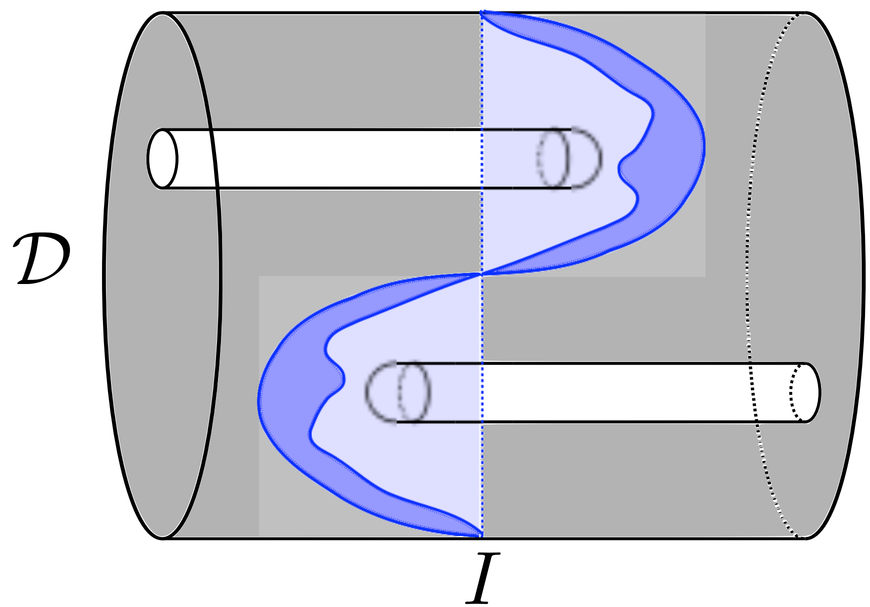

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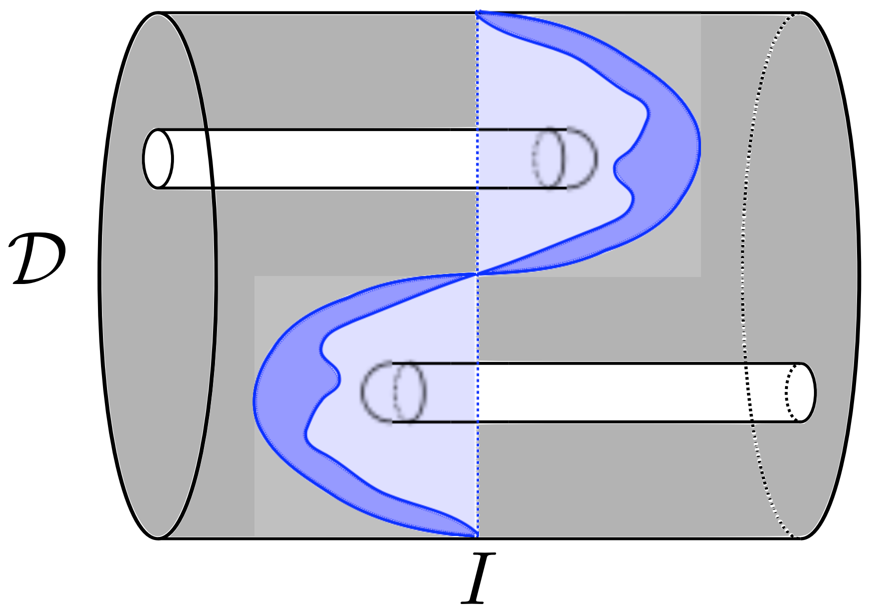

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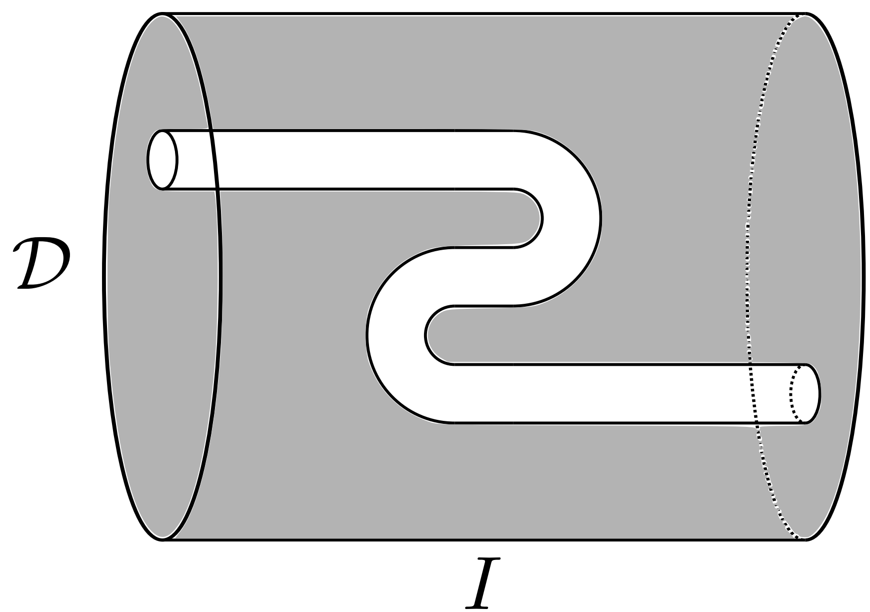
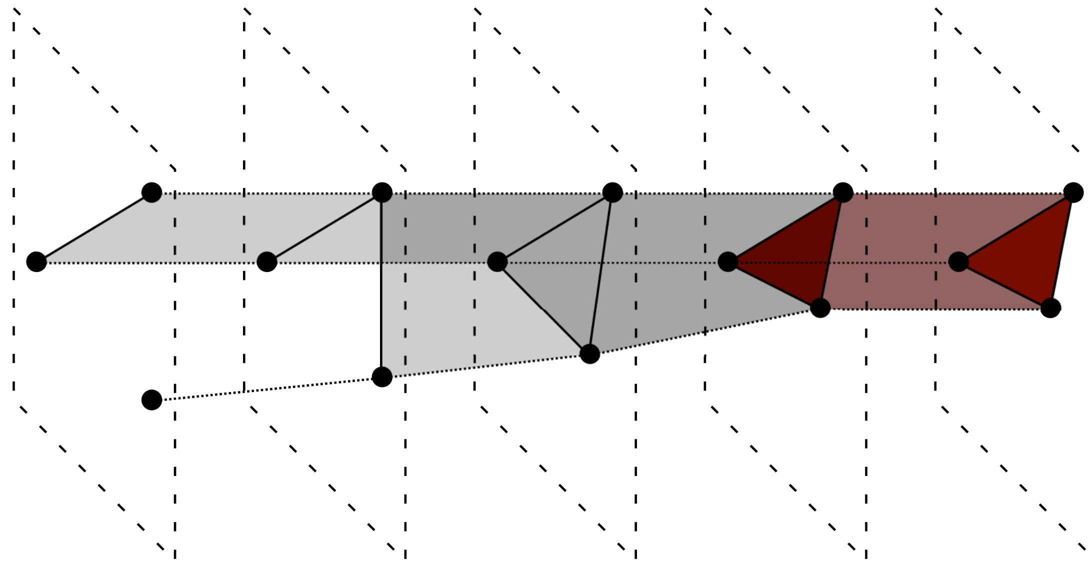


Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology by V. de Silva and R. Ghrist

Zigzag persistence

- Form $Z\mathcal{SC}$ and take $H_{n-1}(Z\mathcal{SC}) \cong H_{n-1}(ZX)$.



Zigzag Persistence by G. Carlsson and V. de Silva

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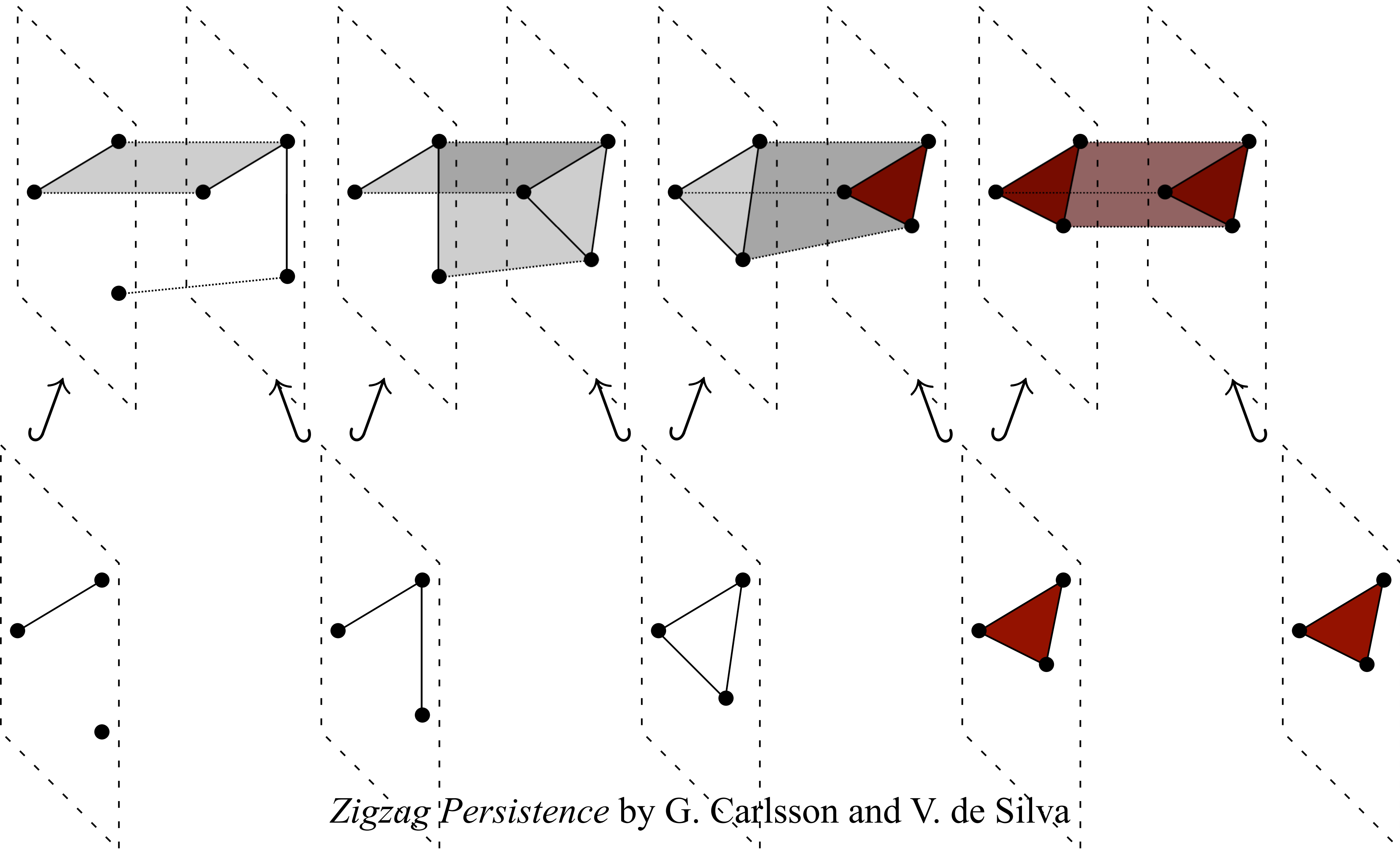

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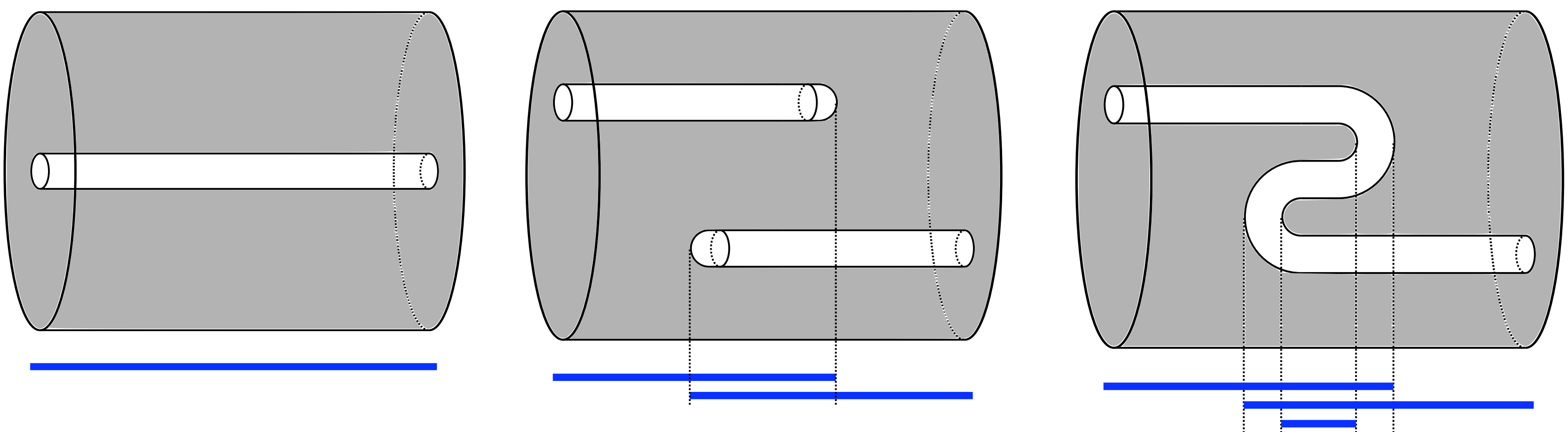

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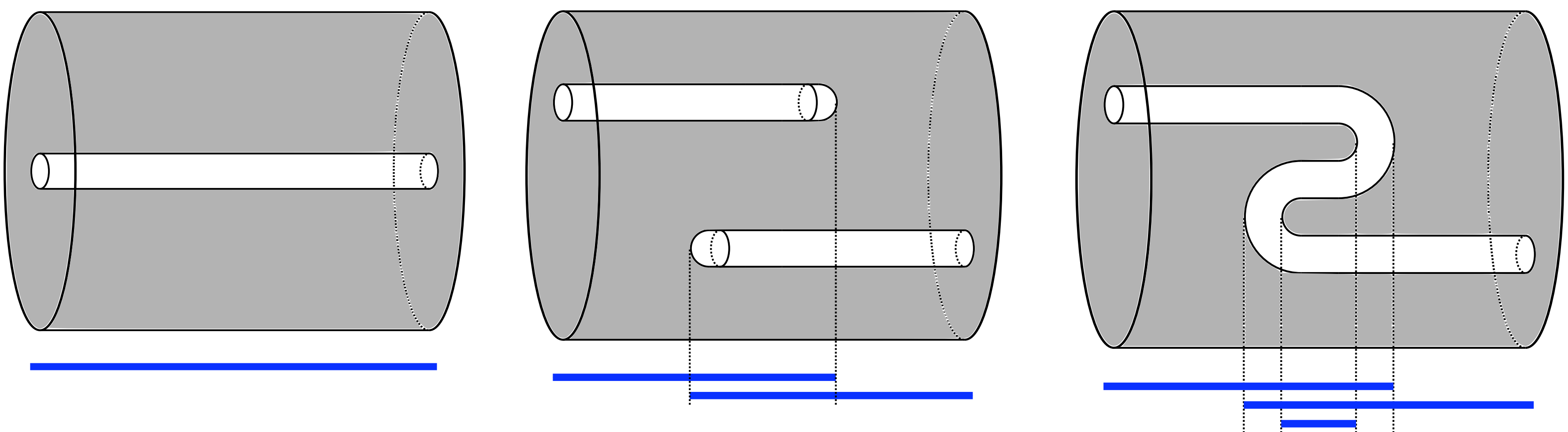

- Hypothesis: there is an evasion path $\Leftrightarrow$ there is a long bar.

Zigzag Persistence by G. Carlsson and V. de Silva

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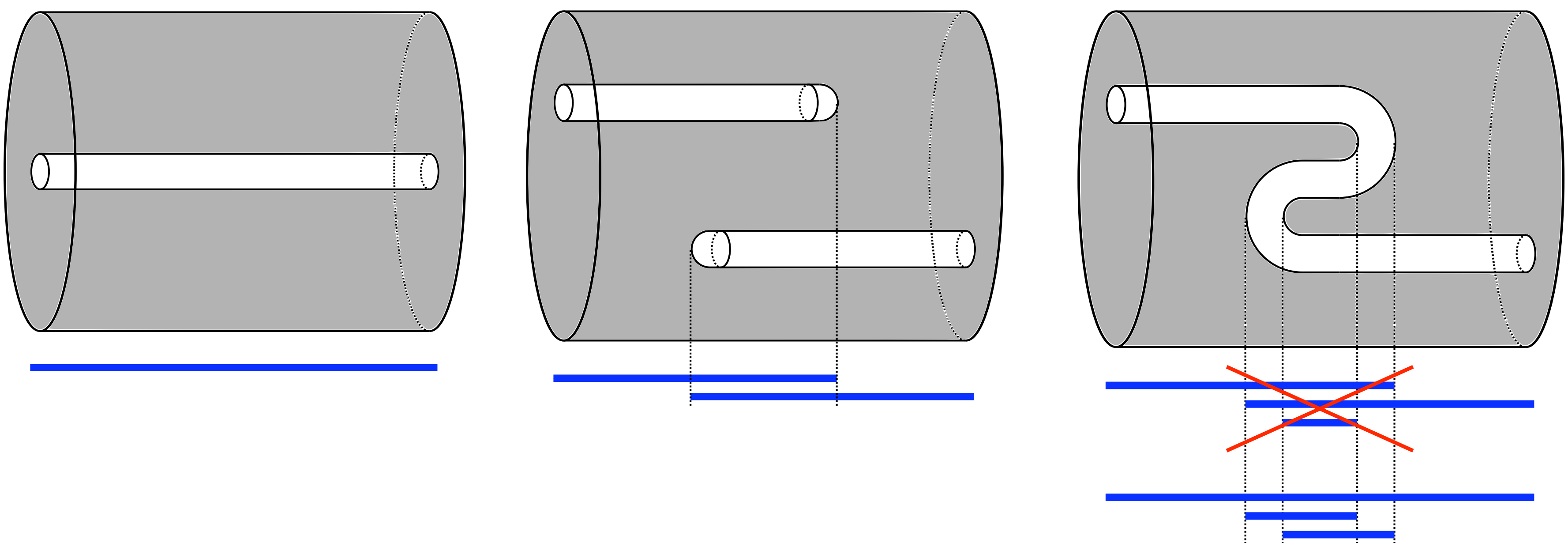

- Hypothesis: there is an evasion path $\Leftrightarrow$ there is a long bar.
- $\Rightarrow$ is true, but $\Leftarrow$ is false.

Zigzag Persistence by G. Carlsson and V. de Silva

Dependence on embedding $X \hookrightarrow \mathcal{D} \times I$

- $\mathcal{SC}$ alone does not determine if an evasion path exists.
- Two networks with the same $\mathcal{SC}$. Top contains evasion path while bottom does not.

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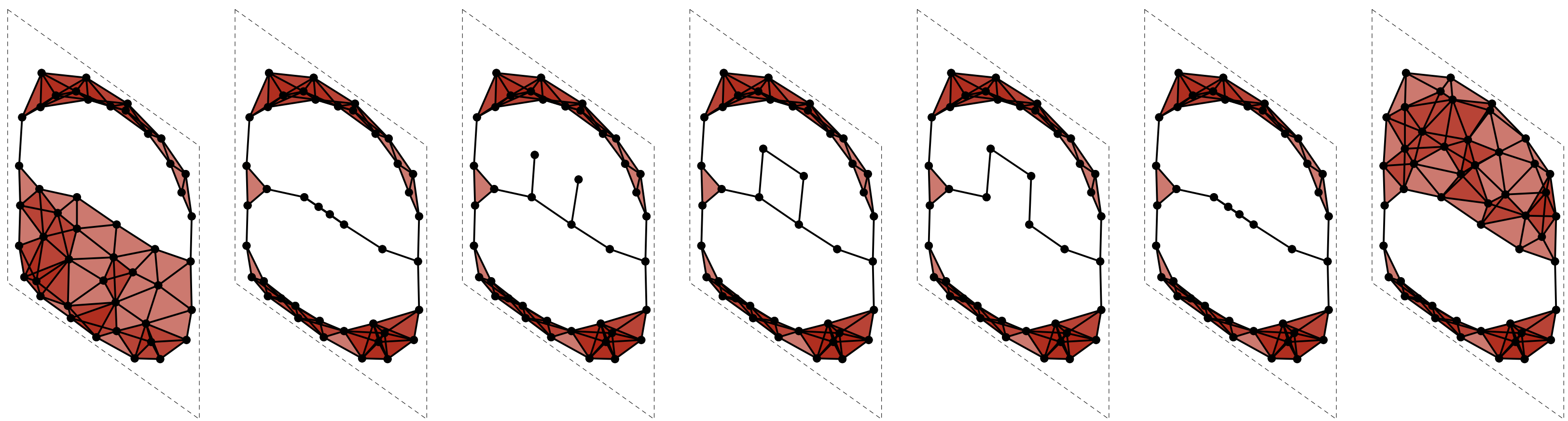

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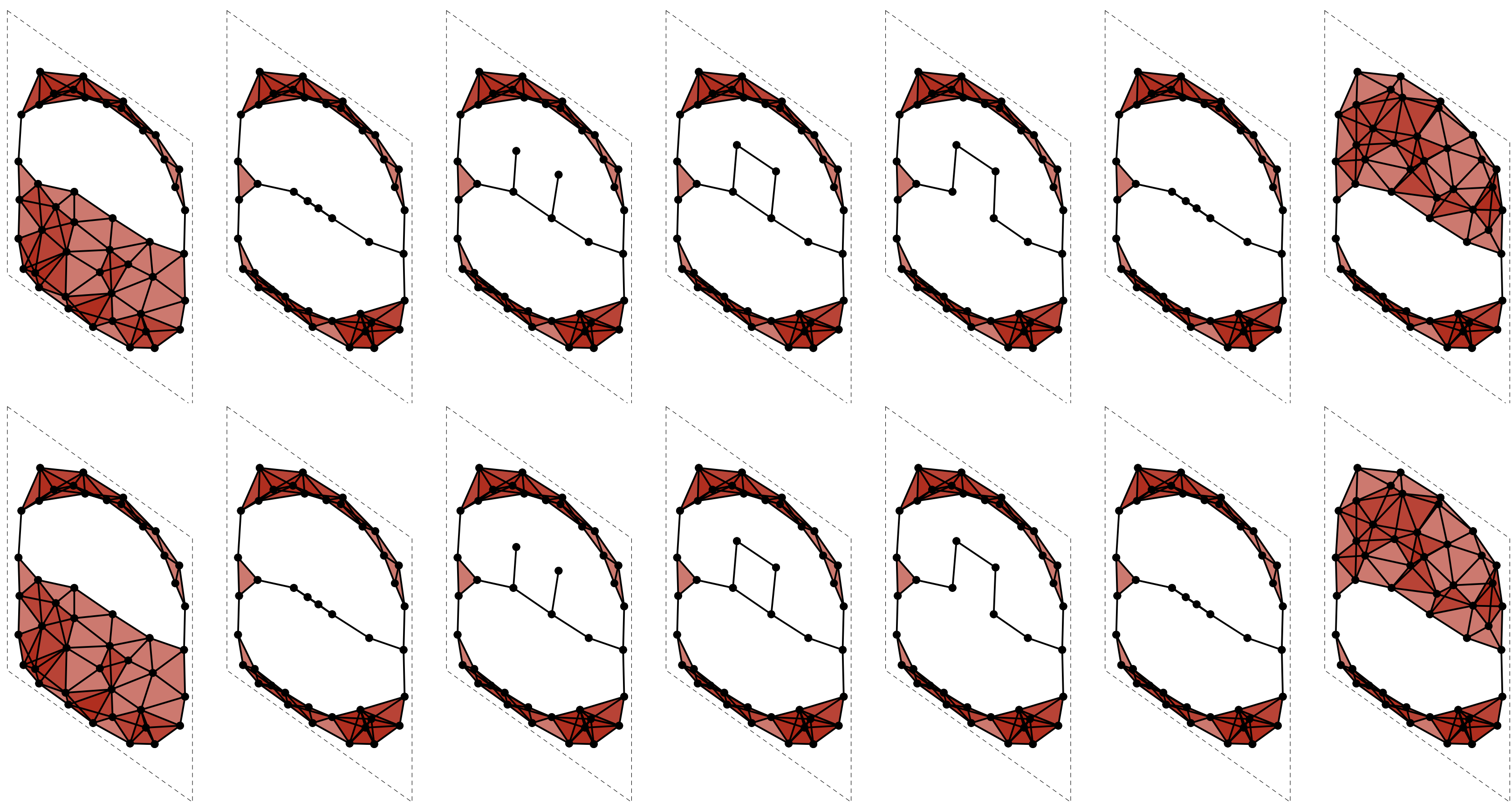

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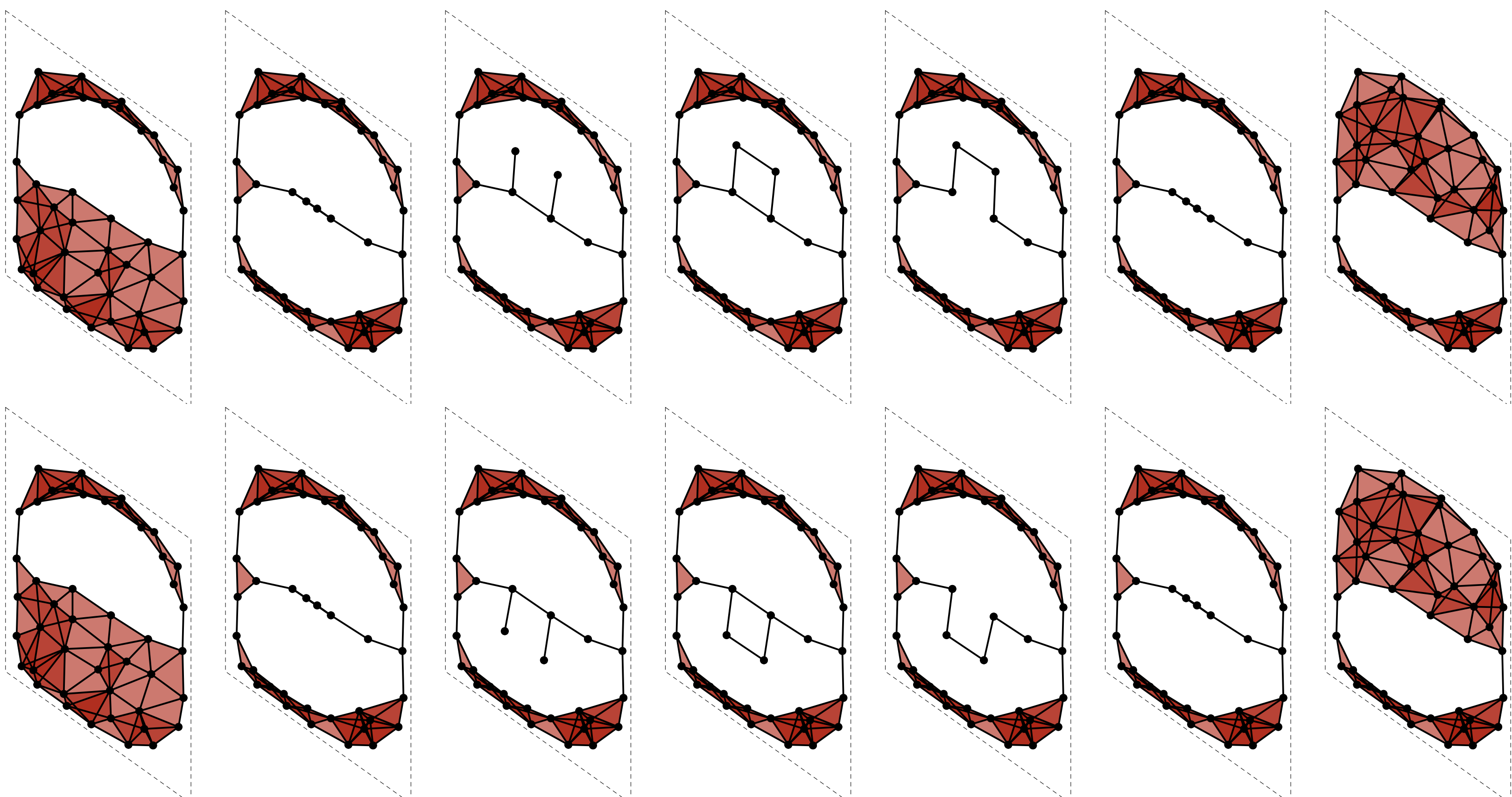

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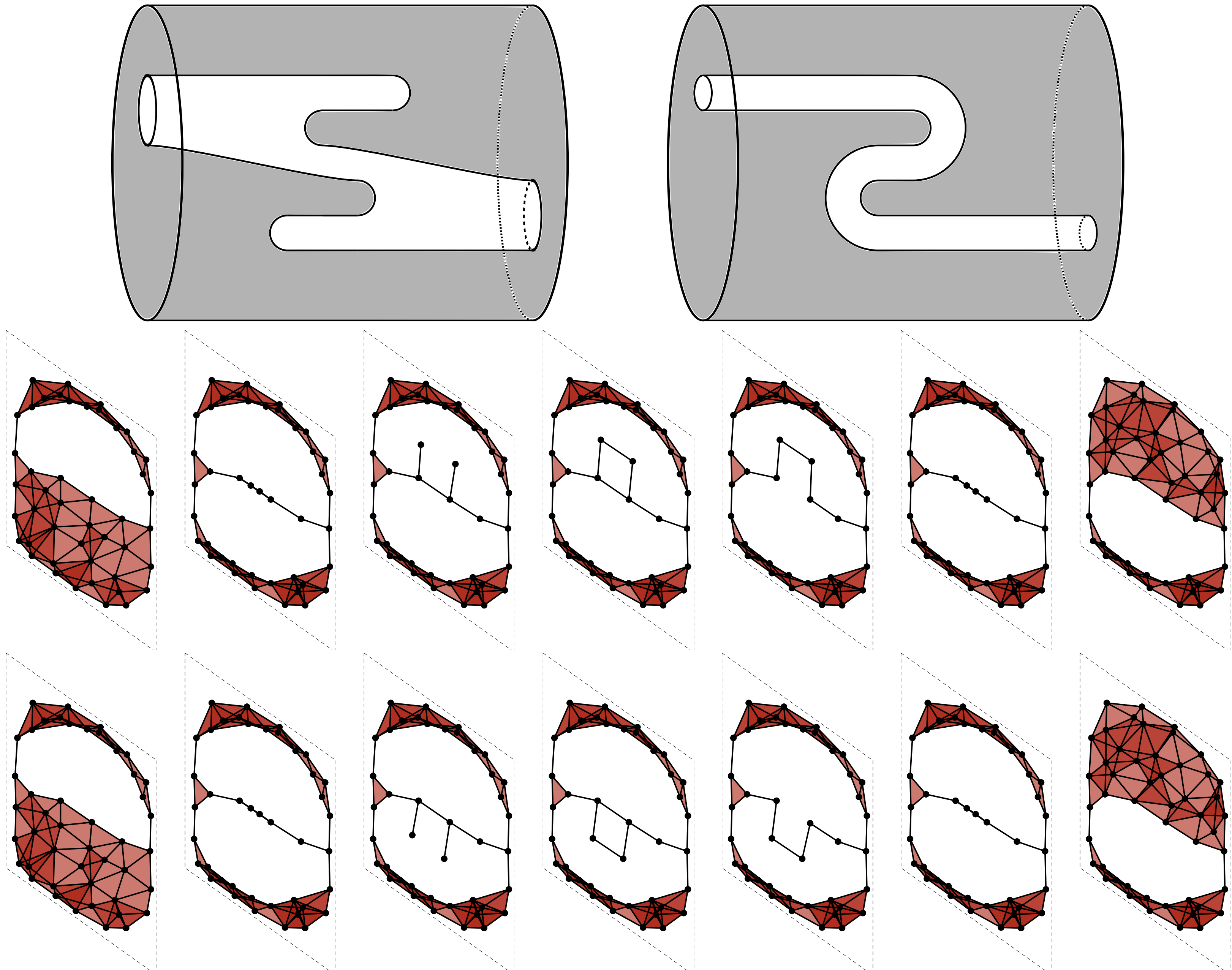

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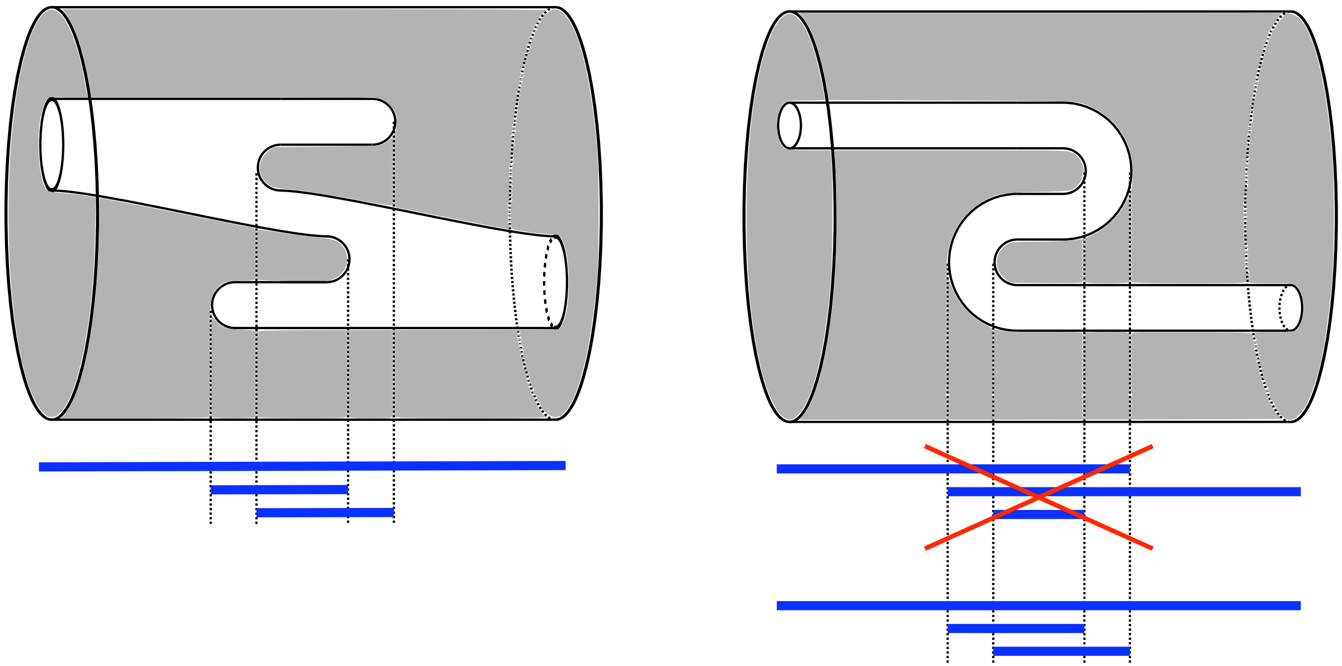
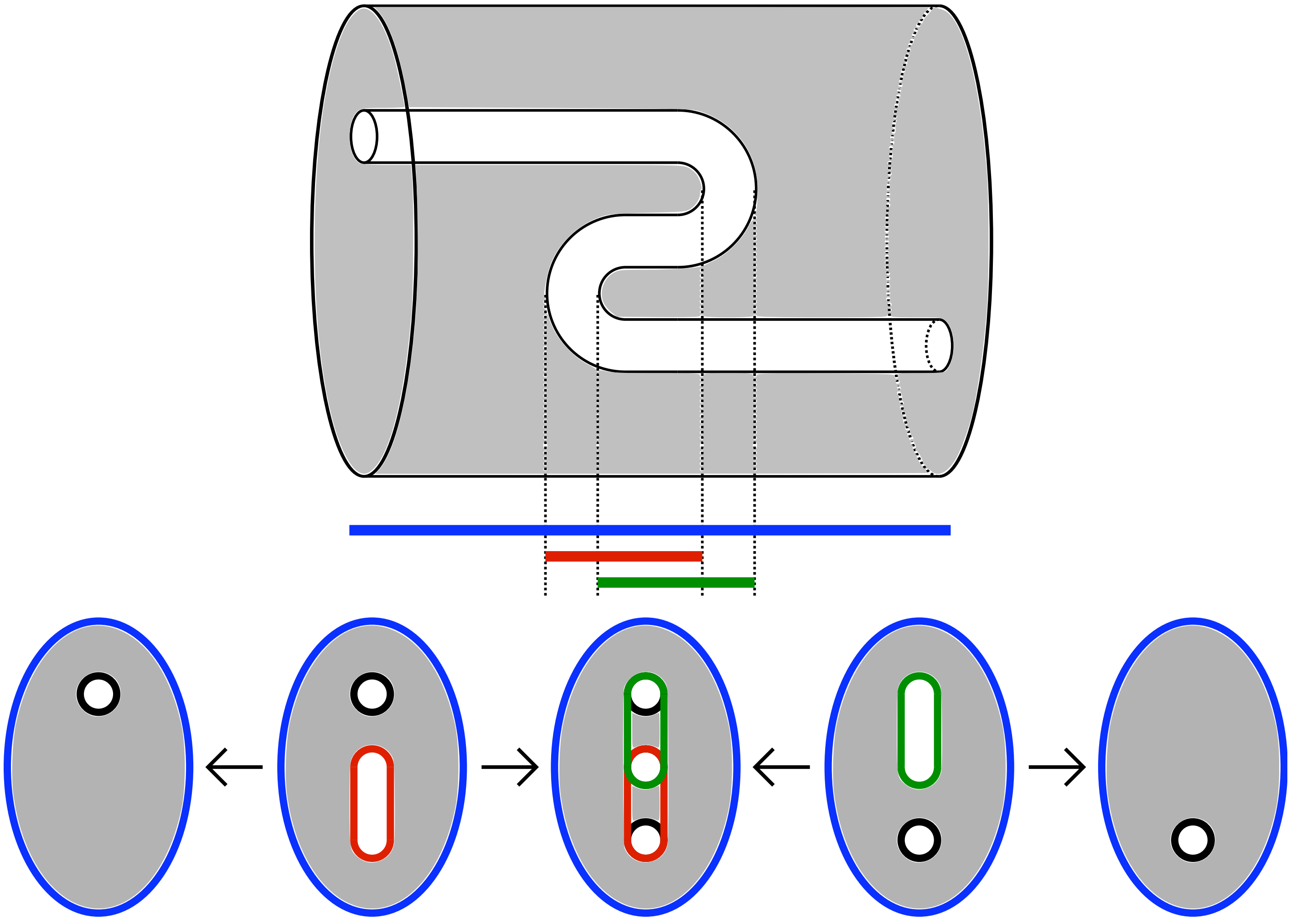


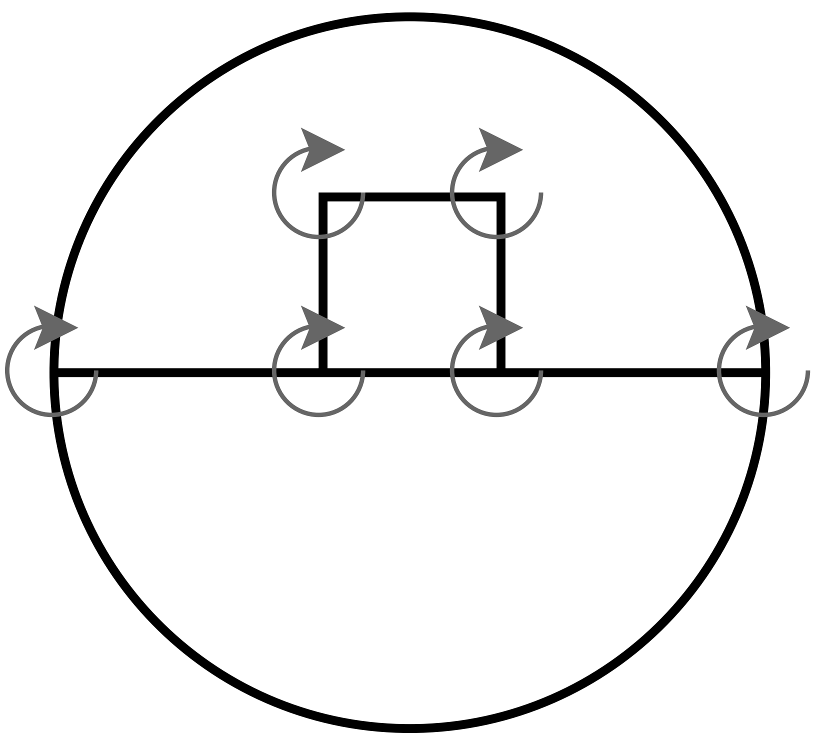
- The two covered regions are fibrewise homotopy equivalent but their complements are not.

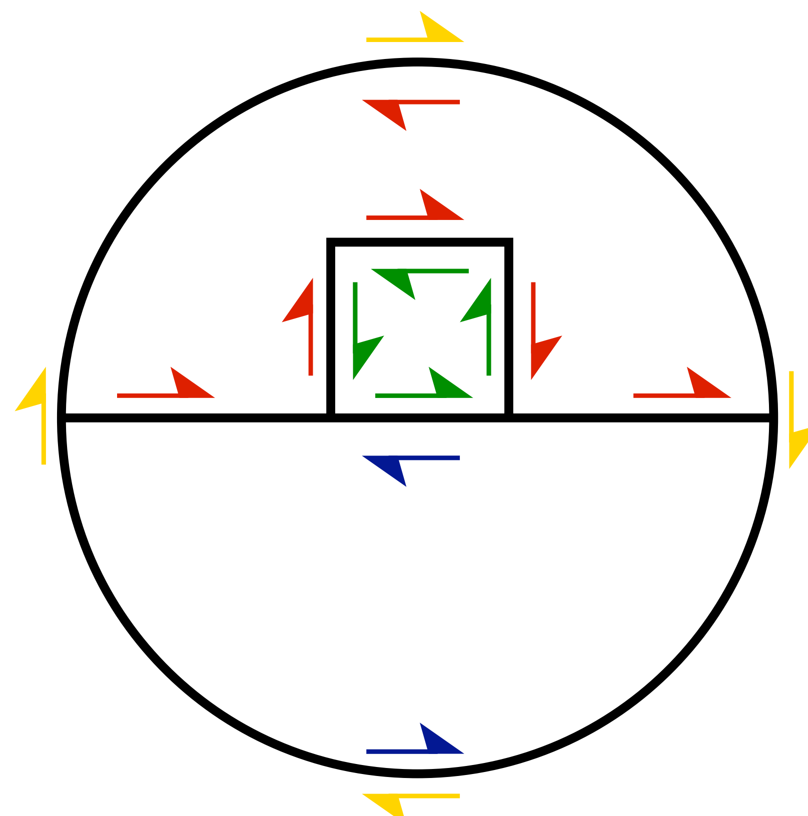
Zigzag persistence



Fat graphs

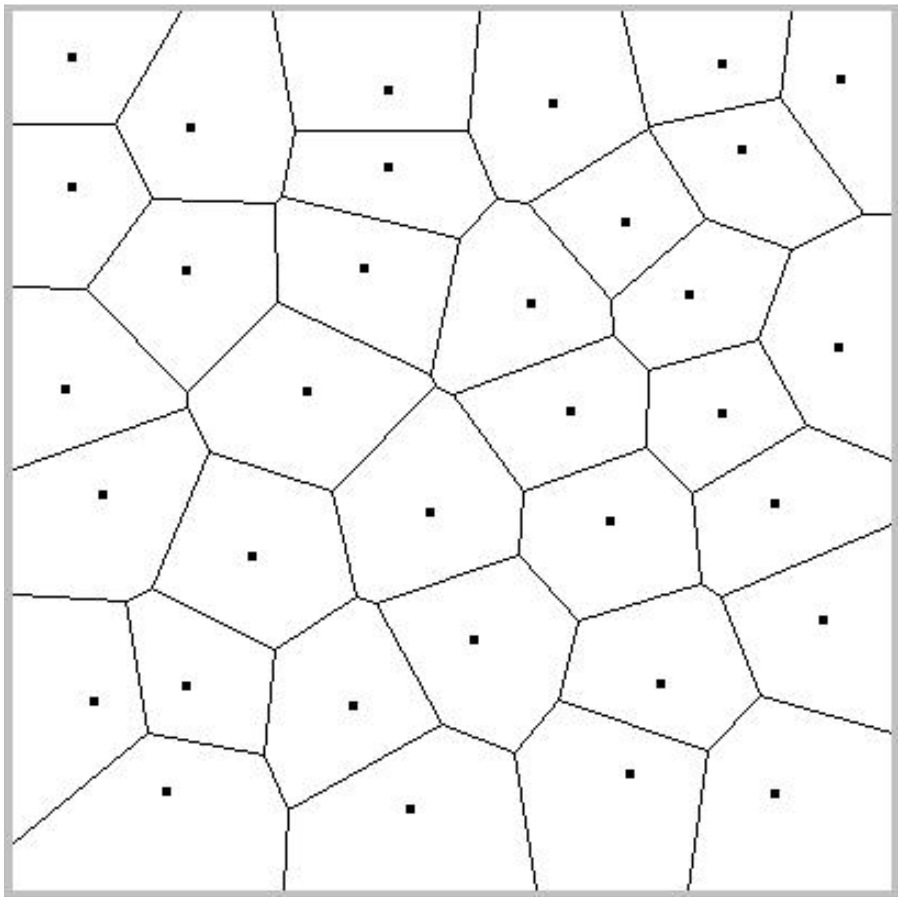
- What minimal sensing capabilities might we add?
- Let $\mathcal{D} \subset \mathbb{R}^2$. A fat graph structure specifies the cyclic ordering of edges adjacent to each vertex.
- Equivalent to a set of boundary cycles.

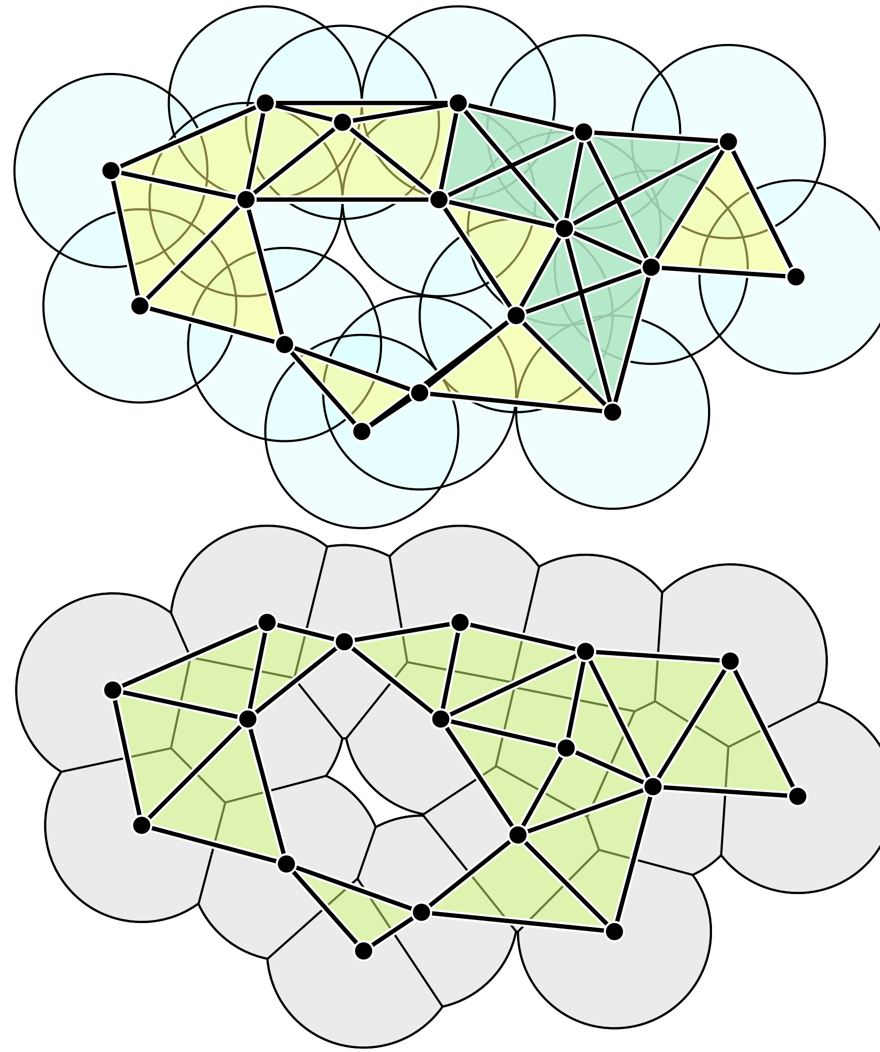




Fat graphs

- The alpha complex and fat graph structure of a connected sensor network determine if an evasion path exists.





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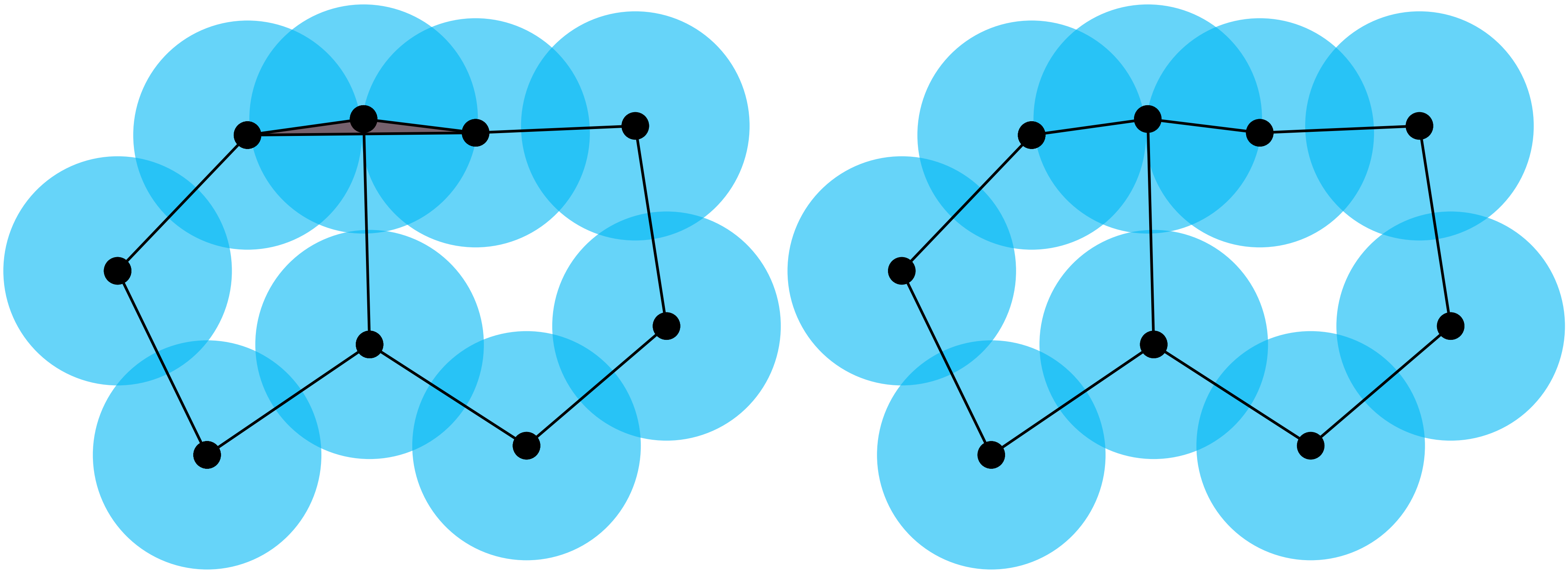

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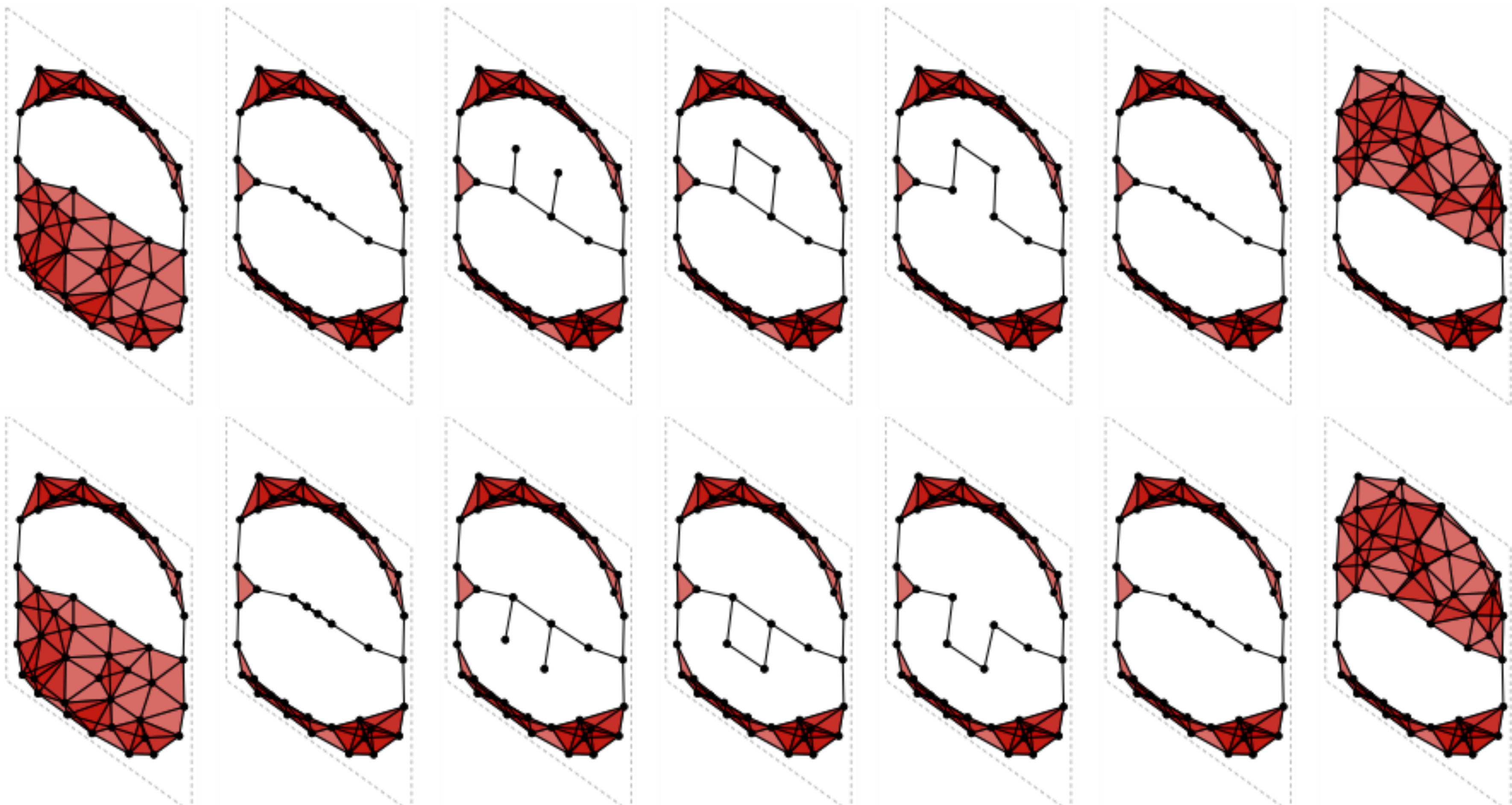

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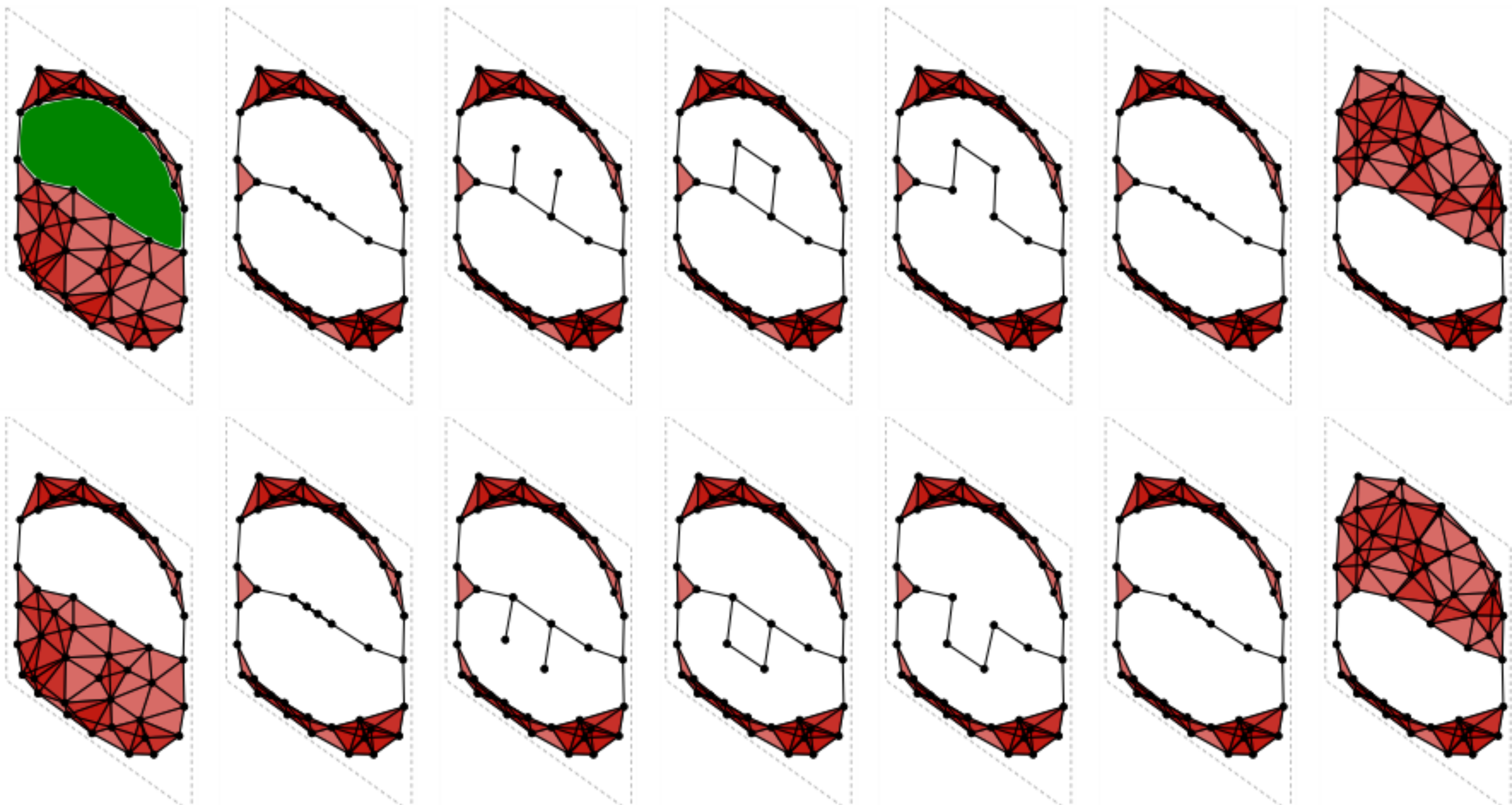

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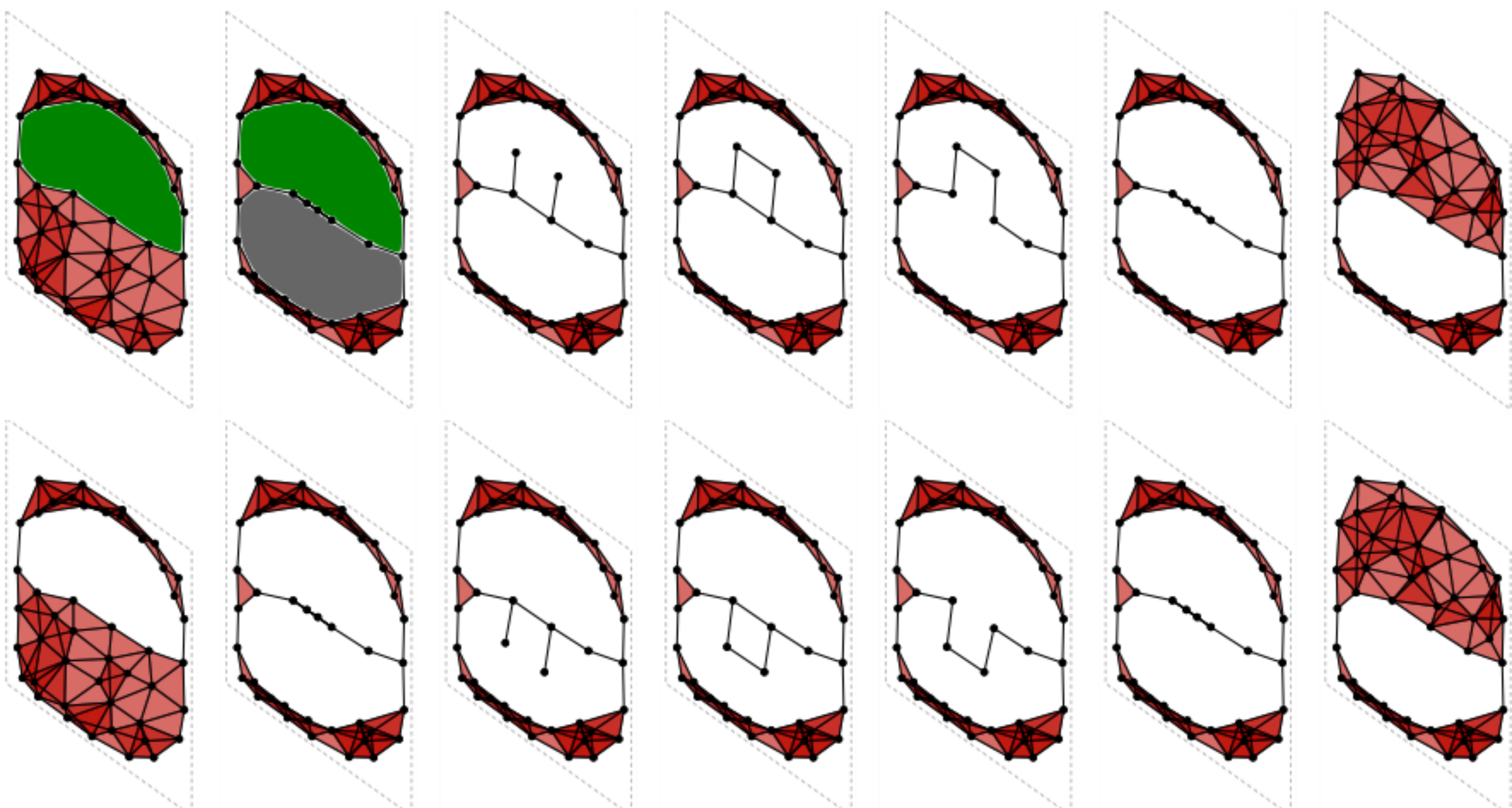

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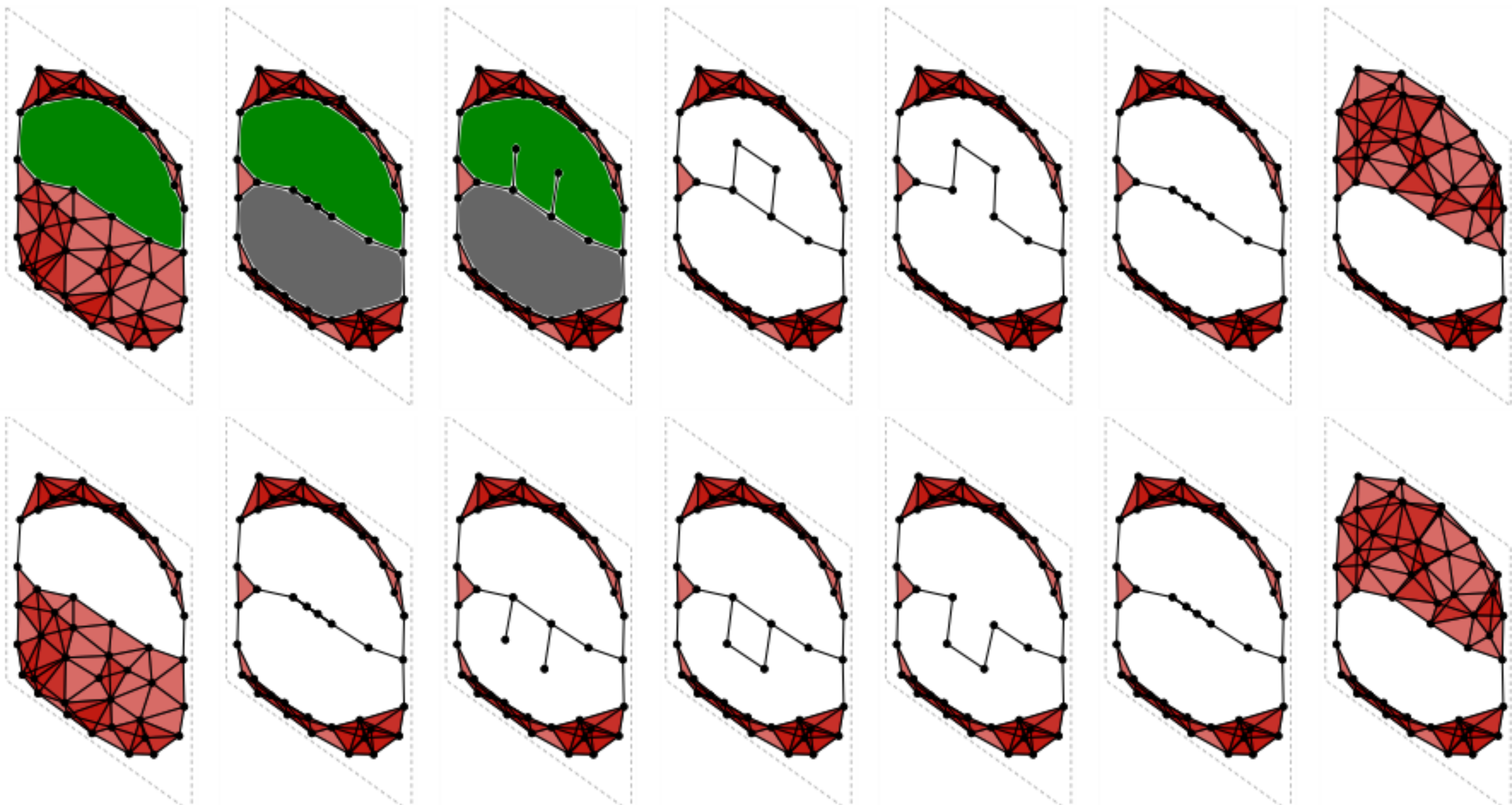

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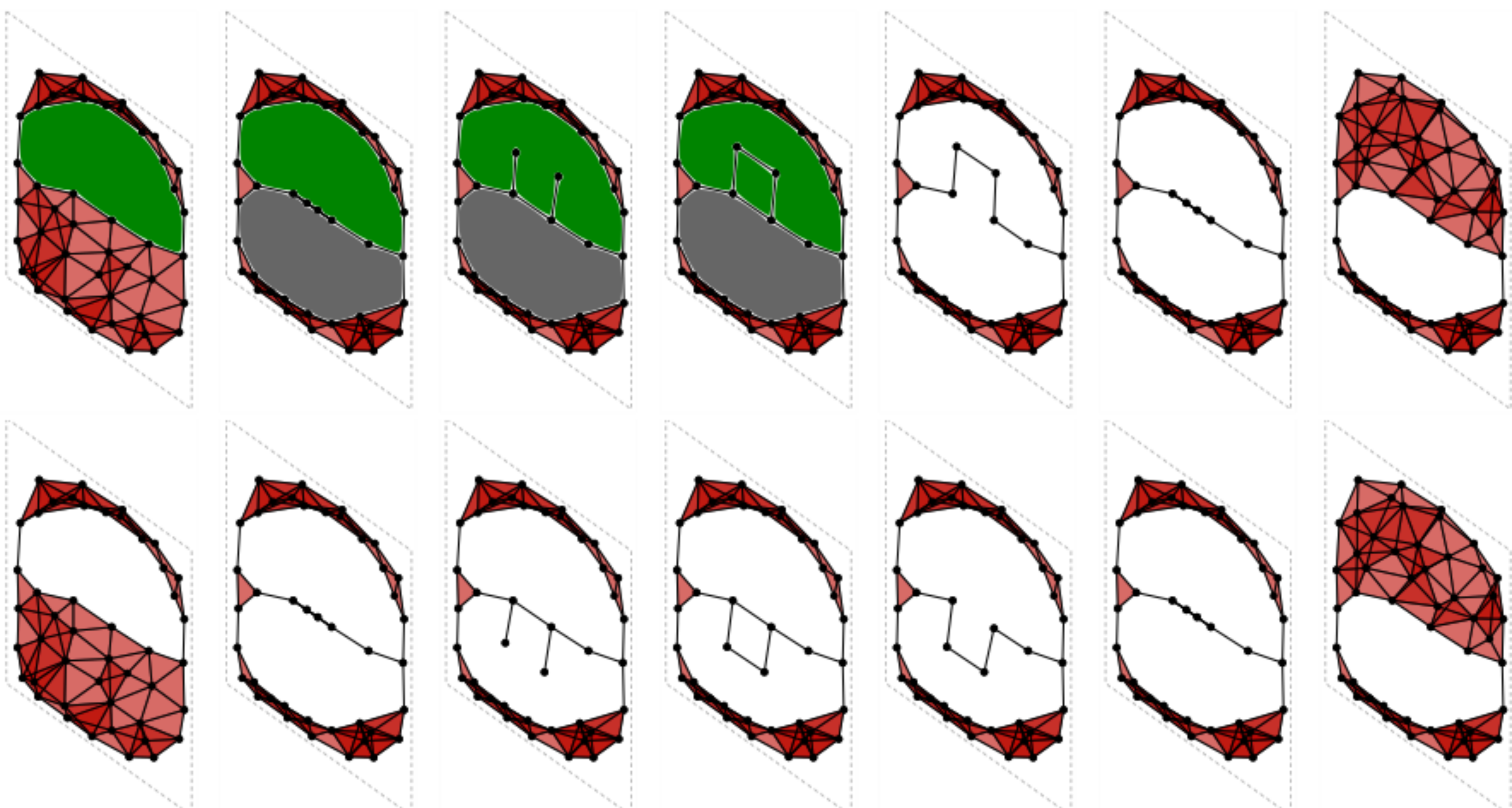

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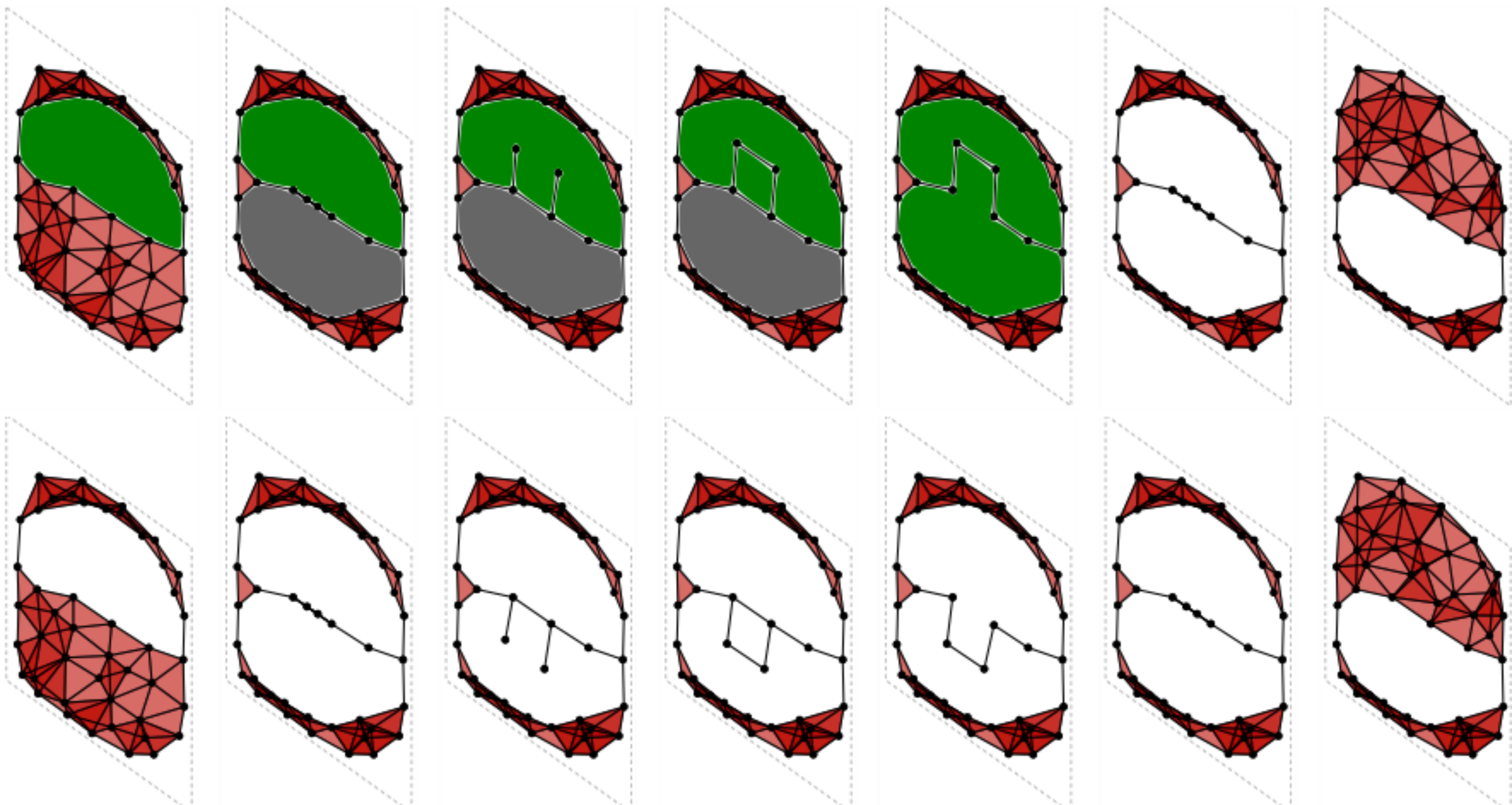

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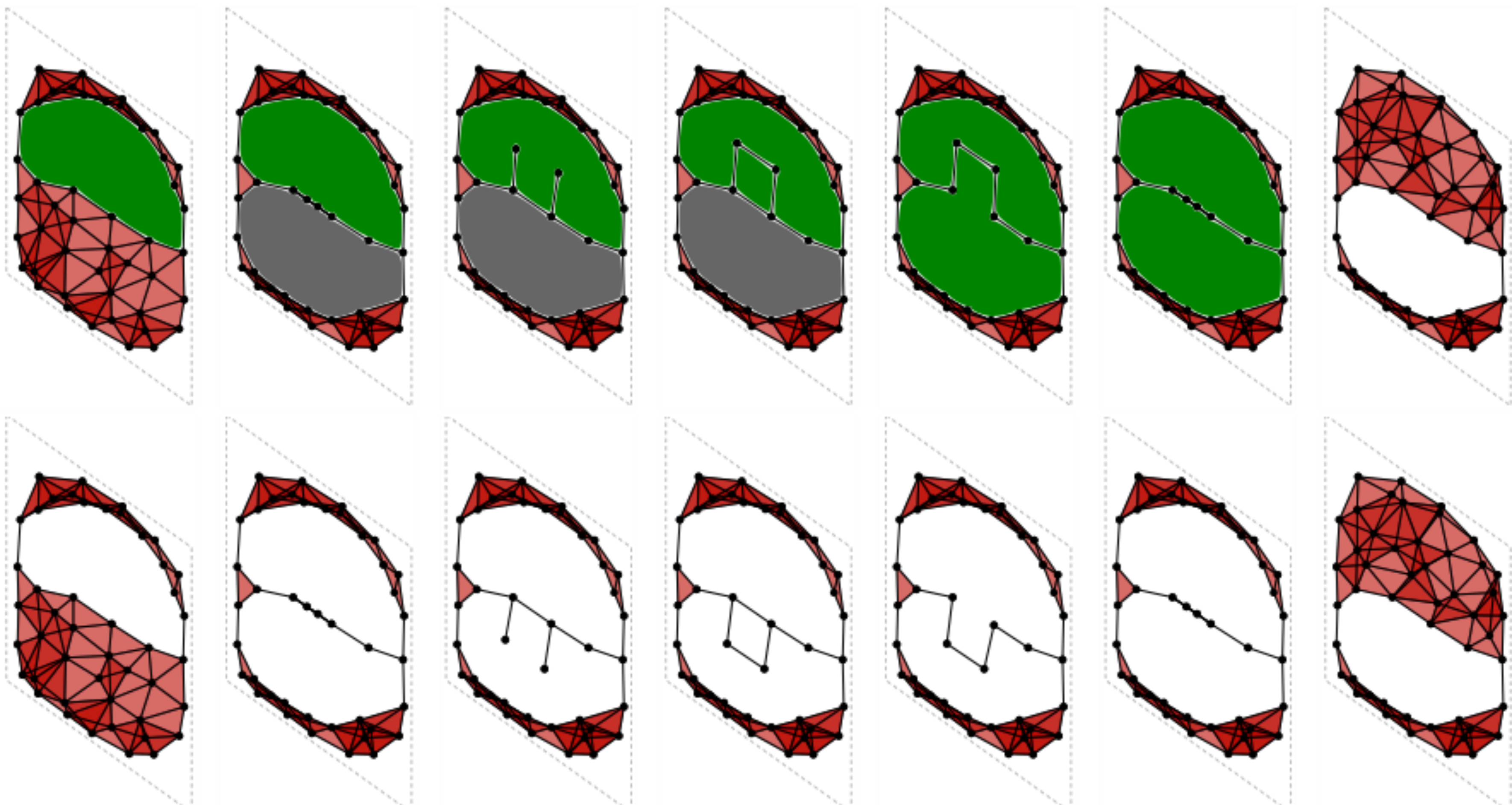

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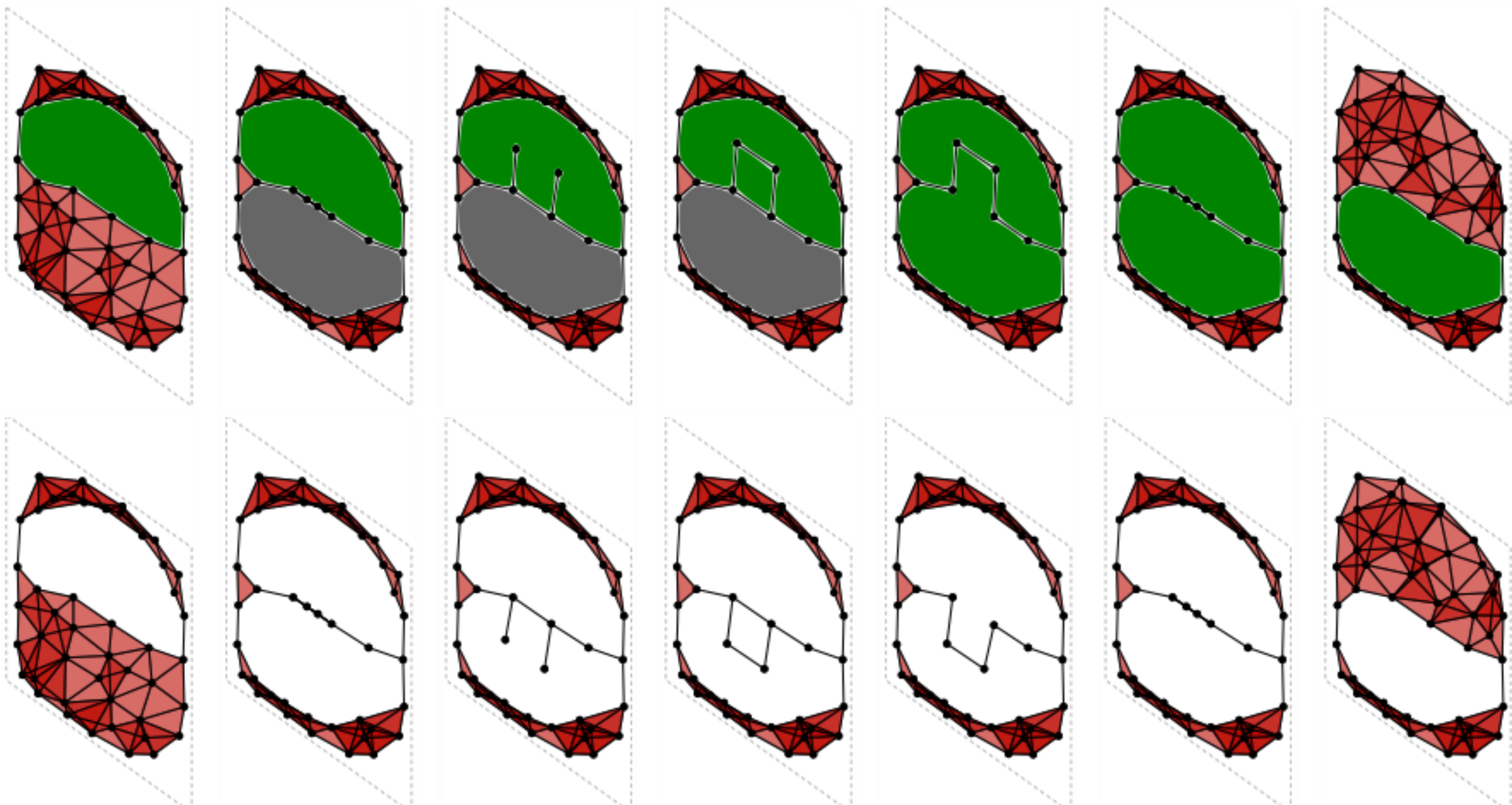

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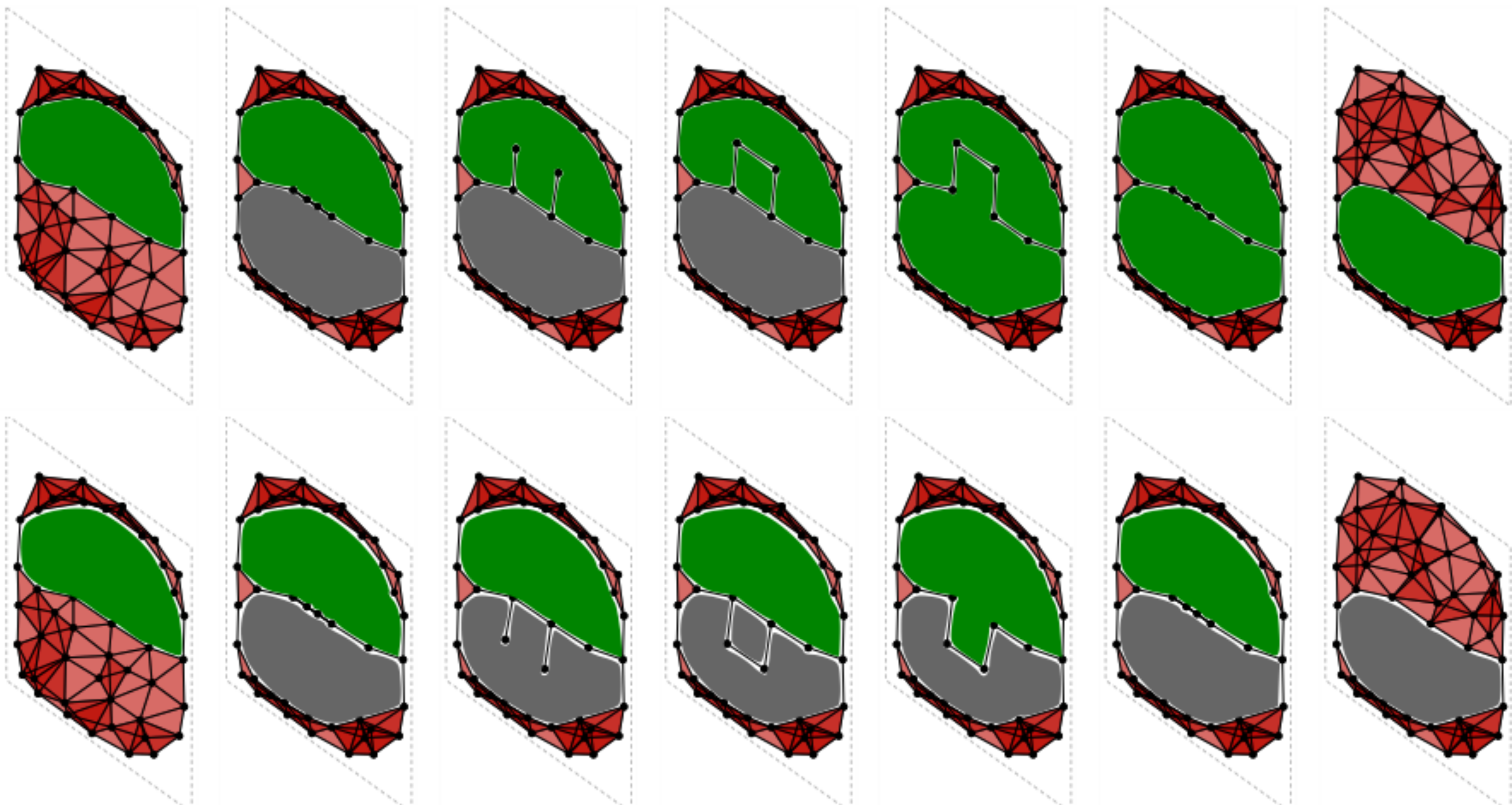

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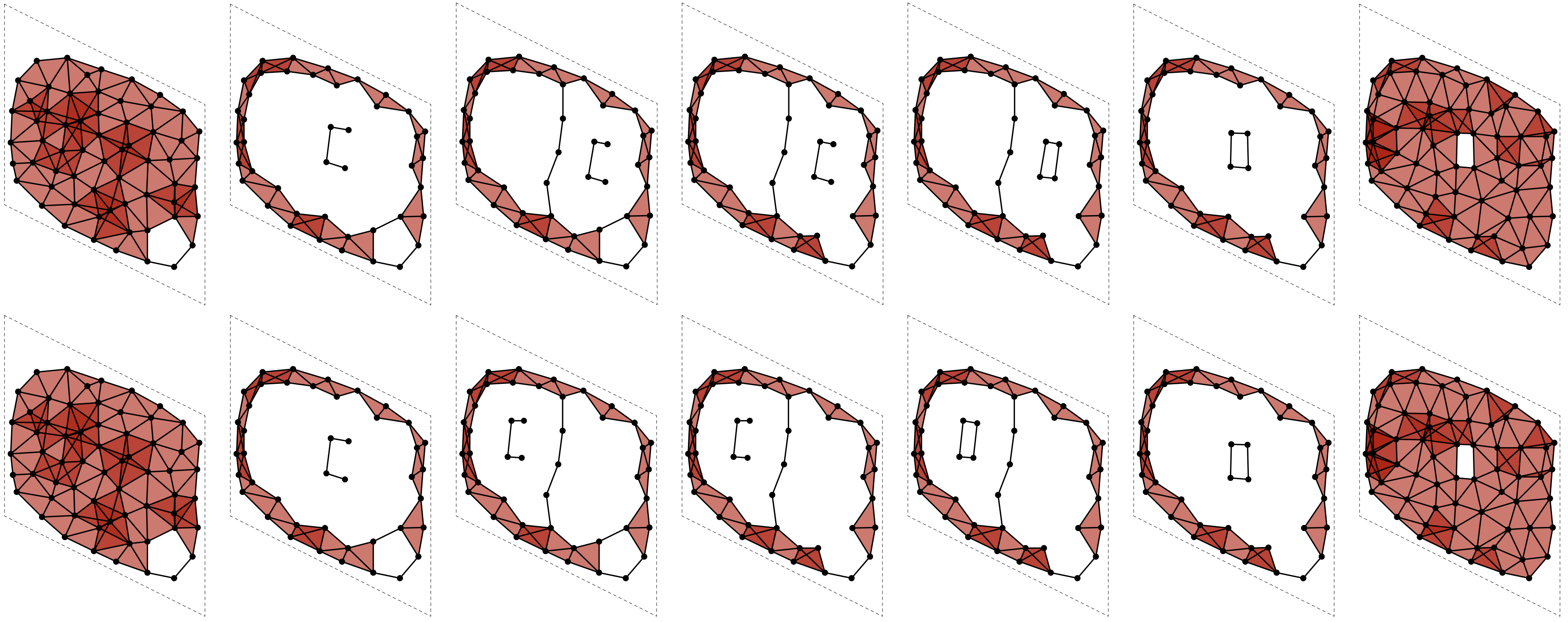

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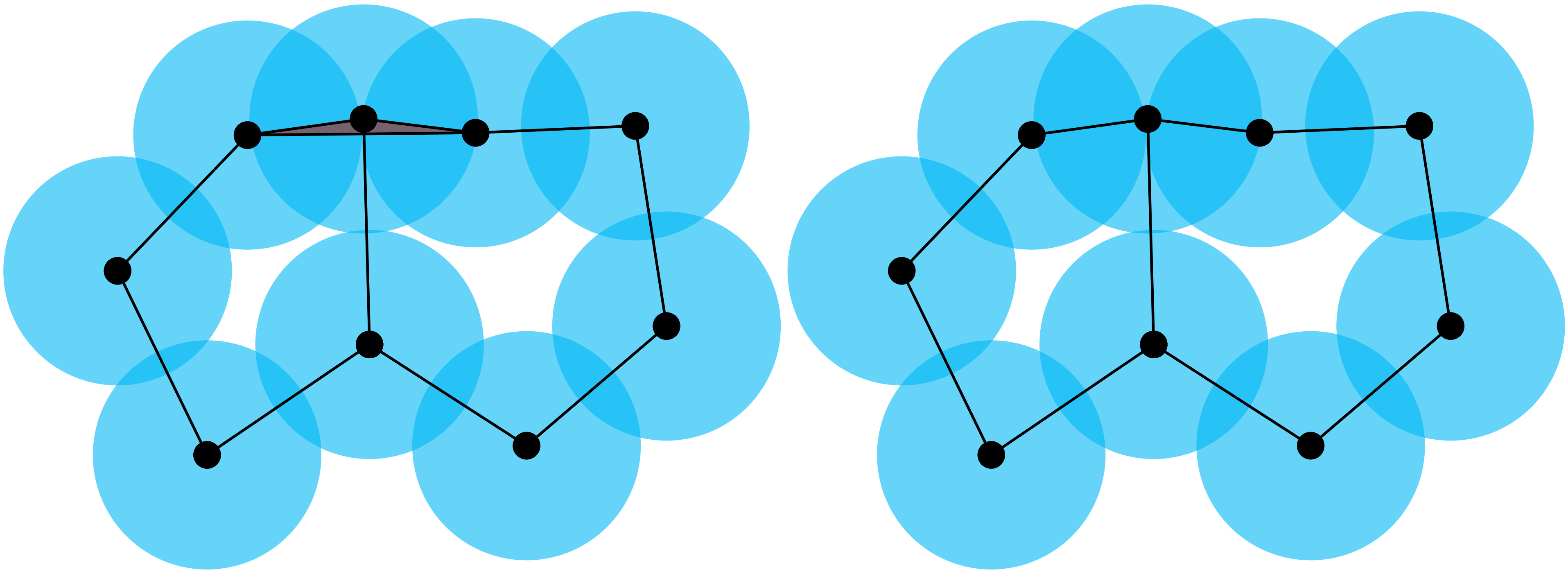
- Are the Čech complex and fat graph structure sufficient?



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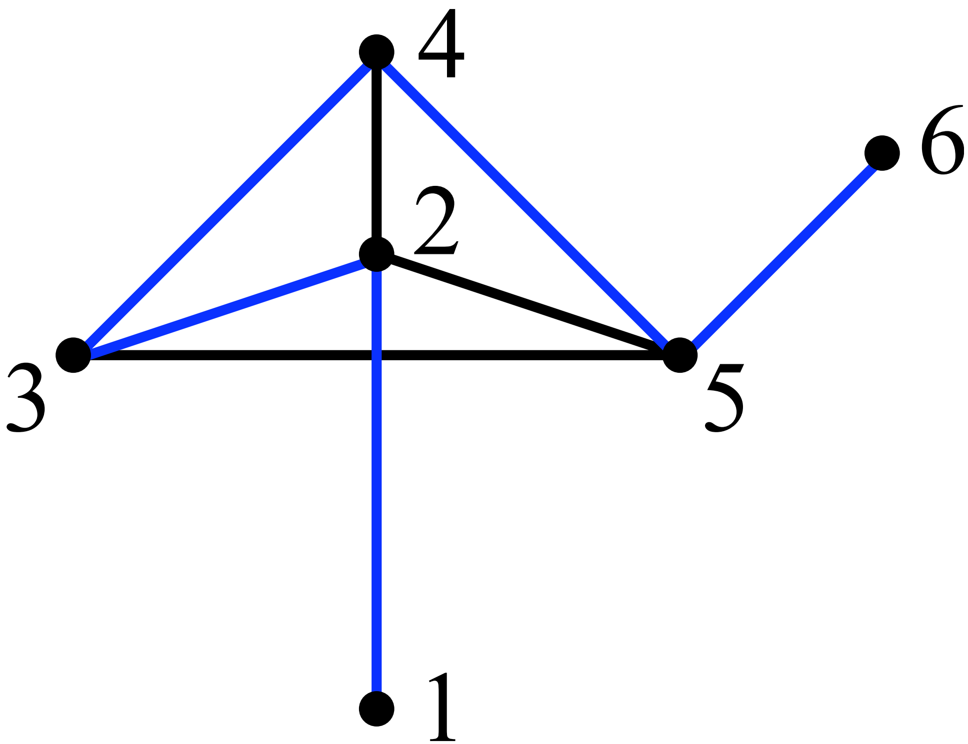
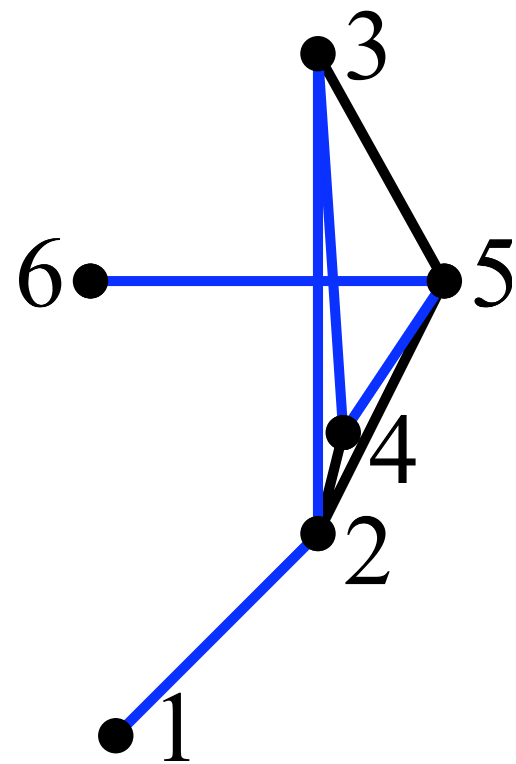
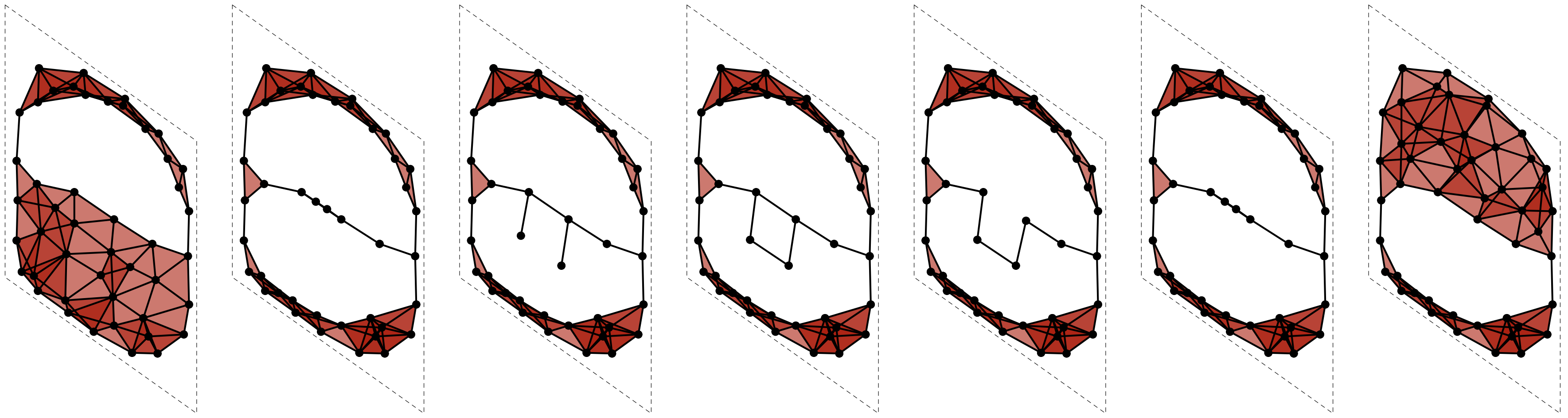




- Final thought: ideas from pure mathematics are often helpful in tackling problems arising from more applied settings.



Thank you!