Applied Topology:
From Global to Local

Henry Adams
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Joint with Michael Moy
From persistent homology to machine learning

Persistent homology measures both the global topology and the local geometry of a dataset.
Global topology

- Left: $H_0$: rank 6, $H_1$: rank 0
- Middle: $H_0$: rank 1, $H_1$: rank 0, $H_2$: rank 1
- Right: $H_0$: rank 1, $H_1$: rank 2, $H_2$: rank 1
Global topology
Global topology

Topology of cyclo-octane energy landscape
Martin, Thompson, Contsias, Watson, 2010
Global topology

Topology of cyclo-octane energy landscape
Martin, Thompson, Coutsias, Watson, 2010
Global topology

Topology of cyclo-octane energy landscape
Martin, Thompson, Contsias, Watson, 2010
Global topology

On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008
Global topology

On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008
Local geometry

Measures of order for nearly hexagonal lattices
Motta, Neville, Shipman, Pearson, Bradley, 2018
Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014
Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skewer, 2014
Local geometry

Collective motion, self-organization
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015
Local geometry

Analysis of Kolmogorov flow and Rayleigh–Bénard convection using persistent homology
Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischaikow, 2016
Local geometry

Understanding diffusion patterns of glassy, liquid and amorphous materials via persistent homology analysis
Onodera, Kohara, Tahara, Masuno, Inoue, Shiga, Hirata, Tsuchiya, Hiraoka, Obayashi, Ohara, Mizuno, Sakata, 2019
Local geometry

Statistical topological data analysis using persistence landscapes
Bubeník, 2015
Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

Answer: (from left) \( u = 1, 1.75, 2, 1.75, 2, 2 \).

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

Hyperbolic disk  |  Flat disk  |  Disk on sphere

Persistent homology detects curvature
Bubenik, Hull, Patel, Whittle, 2019
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020

See also work by Robins and MacPherson & Schweinhart
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

On the choice of weight functions for linear representations of persistence diagrams
Divol and Polonik, 2019
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Local geometry

Cellular automata simulation of grain growth.

Jeremy Mason research group @UC Davis
Local geometry

Knotting probabilities of DNA molecules confined in restricted volumes: DNA knotting in phage capsids
Arsuaga, Vázquez, Trigueros, Summers, Roca, 2002