An Introduction to Applied Topology

Henry Adams
Colorado State University
An Introduction to Applied Topology

Thanks to Murray State and the organizers: Susanne D’Angelo, Dubravko Ivanšić, Ted Porter, Tim Schroeder, and all special session organizers.
AATRN: 1 or 2 live talks per week
YouTube channel: 2,200 subscribers
20 hours watched per day
Datasets have shapes

Example: Diabetes study
145 points in 5-dimensional space

An attempt to define the nature of chemical diabetes using a multidimensional analysis by G. M. Reaven and R. G. Miller, 1979
Datasets have shapes

Example: Cyclo-Octane (C$_8$H$_{16}$) data
1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data by Shawn Martin and Jean-Paul Watson, 2010.
Datasets have shapes

Example: Cyclo-Octane (C₈H₁₆) data
1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Datasets have shapes
A donut and coffee mug are “homotopy equivalent”, and considered to be the same shape. You can bend and stretch (but not tear) one to get the other.
Homology

- $i$-dimensional homology $H_i$ “counts the number of $i$-dimensional holes”
- $i$-dimensional homology $H_i$ actually has the structure of a vector space!

0-dimensional homology $H_0$: rank 6
1-dimensional homology $H_1$: rank 0

0-dimensional homology $H_0$: rank 1
1-dimensional homology $H_1$: rank 3

0-dimensional homology $H_0$: rank 1
1-dimensional homology $H_1$: rank 6
Homology

- $i$-dimensional homology “counts the number of $i$-dimensional holes”
- $i$-dimensional homology actually has the structure of a vector space!

0-dimensional homology $H_0$: rank 1
1-dimensional homology $H_1$: rank 0
2-dimensional homology $H_2$: rank 1

0-dimensional homology $H_0$: rank 1
1-dimensional homology $H_1$: rank 2
2-dimensional homology $H_2$: rank 1

Be careful! (Same as torus over $\mathbb{Z}/2\mathbb{Z}$)

Image credit: https://plus.maths.org/content/imaging-maths-inside-klein-bottle
Topology studies shapes

What shape is this?
Definition

For metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has vertex set $X$ and finite simplex when $\text{diam}(X) \leq r$. 
Definition
For metric space $X$ and scale $r \geq 0$, the \textit{Vietoris–Rips simplicial complex} $\text{VR}(X; r)$ has vertex set $X$ finite simplex when $\text{diam}(X) \leq r$. 
Definition

For metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has vertex set $X$ and finite simplex when $\text{diam}(X) \leq r$. 

Cech_graphics.nb
Definition

For metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has vertex set $X$ and a finite simplex when $\text{diam}(X) \leq r$. 
Definition

For a metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has a finite simplex when $\text{diam}(X) \leq r$. 
Definition

For metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has a finite simplex when $\text{diam}(X) \leq r$. 

![Diagram of Vietoris–Rips simplicial complex](image)
Definition
For metric space \( X \) and scale \( r > 0 \), the Vietoris–Rips simplicial complex \( \text{VR}(X; r) \) has vertex set \( X \) finite simplex when \( \text{diam}(X) \leq r \).
Definition
For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the Čech simplicial complex $\check{\text{Čech}}(X; r)$ has
- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\bigcap_{i=0}^k B(x_i, r) \neq \emptyset$. 

\[Cech_graphics.nb\]
Definition

For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the Čech simplicial complex $\check{Cech}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\bigcap_{i=0}^{k} B(x_i, r) \neq \emptyset$. 
Definition
For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the Čech simplicial complex $\check{\text{Cech}}(X; r)$ has
- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\cap_{i=0}^k B(x_i, r) \neq \emptyset$. 
Definition

For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the 
Čech simplicial complex Čech($X; r$) has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\bigcap_{i=0}^{k} B(x_i, r) \neq \emptyset$. 
Definition

For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the Čech simplicial complex $\check{\text{C}}ech(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\bigcap_{i=0}^{k} B(x_i, r) \neq \emptyset$. 
Definition

For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the Čech simplicial complex $\check{\text{Čech}}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\cap_{i=0}^k B(x_i, r) \neq \emptyset$. 
Definition

For a data set $X \subseteq \mathbb{R}^n$ and scale $r \geq 0$, the \textit{Čech simplicial complex} $\check{\text{Čech}}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $\bigcap_{i=0}^{k} B(x_i, r) \neq \emptyset$. 
Nerve Lemma. \( \check{\text{Cech}}(X; r) \simeq \text{union of balls} \)

Definition

For a data set \( X \subseteq \mathbb{R}^n \) and scale \( r \geq 0 \), the \textit{Čech simplicial complex} \( \check{\text{Cech}}(X; r) \) has

- vertex set \( X \)
- finite simplex \( \{x_0, x_1, \ldots, x_k\} \) when \( \bigcap_{i=0}^{k} B(x_i, r) \neq \emptyset \).
Definition

For a metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $VR(X; r)$ has

- vertex set $X$
- finite simplex \{${x_0, x_1, \ldots, x_k}$\} when $d(x_i, x_j) \leq r$ for all $i, j$. 
Definition

For a metric space $X$ and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\text{VR}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 
Definition

For a metric space $X$ and scale $r \geq 0$, the **Vietoris–Rips simplicial complex** $\text{VR}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 
Definition

For a metric space $X$ and scale $r \geq 0$, the Vietoris–Rips simplicial complex $\text{VR}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 
Definition

For a metric space \( X \) and scale \( r \geq 0 \), the *Vietoris–Rips simplicial complex* \( VR(X; r) \) has

- vertex set \( X \)
- finite simplex \( \{x_0, x_1, \ldots, x_k\} \) when \( d(x_i, x_j) \leq r \) for all \( i, j \).
Definition

For a metric space $X$ and scale $r \geq 0$, the \textit{Vietoris–Rips simplicial complex} $VR(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 
Definition

For a metric space $X$ and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\text{VR}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 

Python Code:

```python
In[73]:=

Demo[data1, 0, .41]

Out[73]=

Cech simplicial complex
Appearance
draw one simplices
draw Cech complex
draw Rips complex
Filtration parameter

0.338

CechRips.nb

7
```
**Definition**

For a metric space $X$ and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\text{VR}(X; r)$ has

- vertex set $X$
- finite simplex $\{x_0, x_1, \ldots, x_k\}$ when $d(x_i, x_j) \leq r$ for all $i, j$. 

Cech simplicial complex
Appearance
draw one simplices
draw Cech complex
draw Rips complex
Filtration parameter $t$

8
CechRips.nb
In this section we describe how to use only a finite sampling from some unknown underlying space to estimate the underlying space's topology. The first step is to build a nested family of simplicial complexes, and the second is to apply persistence to these complexes. Persistent homology depends on the fact that the map from a topological space to its Vietoris–Rips complex is a functor. This means that for any continuous map of topological spaces, the induced map on their Vietoris–Rips complexes is well-defined. Hence, if we have a map \( f: Y \to X \) between topological spaces, we can consider the Vietoris–Rips complexes \( VR(Y) \) and \( VR(X) \) and the induced map \( VR(f): VR(Y) \to VR(X) \).

Let us consider an example. Let \( X \) be 21 points which (unknown to us) are noisily sampled from a circle. Figure 3 contains four nested Vietoris–Rips complexes \( VR(X) \), with \( d \) denoting the radius of the balls defining the complexes. The topological profile of this example, 

\[
\text{Betti numbers are one way of distinguishing between different topological spaces.}
\]

We refer the interested reader to [4, 24] for more information on homology, to [14, 20, 21, 36] for introductions to persistent homology, and to [21, 36] for persistent homology 

\[
\text{in zigzag persistence, the direction of maps along a sequence of topological spaces is arbitrary, as opposed to the unidirectional sequence of maps in classical homology.}
\]

In this section, we describe how to use only a finite sampling from some unknown underlying space to estimate the underlying space's topology. The first step is to build a nested family of simplicial complexes, and the second is to apply persistent homology. This is the same topological approach used to analyze optical range image patches in [2, 16]. We refer the interested reader to [4, 24] for more information on homology.
Persistent homology

- Significant features persist.
- Cubic computation time in the number of simplices.
• Significant features persist.
• Cubic computation time in the number of simplices.
• Significant features persist.
• Cubic computation time in the number of simplices.


**Persistent homology**

**Sublevelset persistence**

- Significant features persist.
- Cubic computation time in the number of simplices.

Analysis of Kolmogorov flow and Rayleigh–Bénard convection using persistent homology by Miroslav Kramár, Rachel Levanger, Jeffrey Tithof, Balachandra Suri, Mu Xu, Mark Paul, Michael F Schatz, Konstantin Mischaikow
• Significant features persist.
• Cubic computation time in the number of simplices.
Zigzag persistence

• **Definition.** A zigzag module over zigzag diagram

  \[ \bullet \leftrightarrow \bullet \leftrightarrow \ldots \leftrightarrow \bullet \leftrightarrow \bullet \]

  has a vector space at each vertex and a linear map at each edge.

• **Example.**

  \[ \bullet \rightarrow \bullet \leftarrow \bullet \rightarrow \bullet \]

  \[ V_1 \rightarrow V_2 \leftarrow V_3 \rightarrow V_4 \]
Zigzag persistence

- **Definition.** A zigzag module over zigzag diagram
  \[
  \bullet \leftrightarrow \bullet \leftrightarrow \ldots \leftrightarrow \bullet \leftrightarrow \bullet
  \]
  has a vector space at each vertex and a linear map at each edge.

- **Example.**
  \[
  \bullet \longrightarrow \bullet \longleftarrow \bullet \longrightarrow \bullet
  \]
  \[
  V_1 \rightarrow V_2 \leftarrow V_3 \rightarrow V_4
  \]
  \[
  U_1 \rightarrow U_2 \leftarrow U_3 \rightarrow U_4
  \]
Zigzag persistence

• **Definition.** A zigzag module over zigzag diagram

  \[
  \bullet \leftrightarrow \bullet \leftrightarrow \ldots \leftrightarrow \bullet \leftrightarrow \bullet
  \]

  has a vector space at each vertex and a linear map at each edge.

• **Example.**

  \[
  \bullet \longrightarrow \bullet \longleftarrow \bullet \longrightarrow \bullet
  \]

  \[
  V_1 \rightarrow V_2 \leftarrow V_3 \rightarrow V_4
  \]

• **Theorem (Gabriel).** Indecomposables classified by intervals.

  \[
  0 \leftrightarrow \ldots \leftrightarrow 0 \leftrightarrow k \leftrightarrow \ldots \leftrightarrow k \leftrightarrow 0 \leftrightarrow \ldots \leftrightarrow 0
  \]

*Zigzag Persistence* by G. Carlsson and V. de Silva
Zigzag persistence

- **Definition.** A zigzag module over zigzag diagram
  \[ \bullet \leftrightarrow \bullet \leftrightarrow \ldots \leftrightarrow \bullet \leftrightarrow \bullet \]
  has a vector space at each vertex and a linear map at each edge.

- **Example.**
  \[ \bullet \rightarrow \bullet \leftarrow \bullet \rightarrow \bullet \]
  \[ V_1 \rightarrow V_2 \leftarrow V_3 \rightarrow V_4 \]

- **Theorem (Gabriel).** Indecomposables classified by intervals.
  \[ 0 \leftrightarrow \ldots \leftrightarrow 0 \leftrightarrow k \leftrightarrow \ldots \leftrightarrow k \leftrightarrow 0 \leftrightarrow \ldots \leftrightarrow 0 \]

*Zigzag Persistence* by G. Carlsson and V. de Silva
Theorem (Gabriel).

Diagram $D$ has a finite number of indecomposables $\iff$ it's a union of certain Dynkin diagrams.

Example 8.

In Example 3, one can identify a 2-dimensional family of pairwise nonisomorphic indecomposable representations, namely, where $V_{a_1}, \ldots, V_{a_5}$ are given by the matrices

\[
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
1 & 1 \\
1 & \lambda \\
1 & \mu
\end{pmatrix},
\]

respectively, with $\lambda, \mu \in K$.

Furthermore, there exist other families of indecomposables for this particular star quiver, where the number of parameters of the family is arbitrarily large. In this example, describing explicitly the set of indecomposable representations is essentially an impossible task.

Theorems of Gabriel and Kac

We have observed different behavior of indecomposables for various quivers. If a quiver has only finitely many indecomposable representations, it is called a quiver of finite type. If there are infinitely many indecomposables, but they appear in families of dimension at most 1, then the quiver is called of tame type. If the representation theory of the quiver is at least as complicated as the representation theory of the double loop quiver, then the quiver is called of wild type. These definitions given here are imprecise but hopefully convey the right intuition. The precise definitions of tame and wild type are omitted. It is known that every quiver is either of finite type, tame, or wild. We will later see that such a trichotomy is true in a more general setting.

Forgetting the orientations of the arrows yields the underlying undirected graph of a quiver. The following amazing theorem is due to Gabriel (see [8], [13]).

Theorem 9 [Gabriel's Theorem, part 1]. A quiver is of finite type if and only if the underlying undirected graph is a union of Dynkin graphs of type $A, D, E$, shown below:

The Dynkin graphs play an important role in the classification of simple Lie algebras, of finite crystallographic root systems and Coxeter groups, and other objects of "finite type".

For quivers of tame type, a similar description exists, namely:

Theorem 10 ([5], [14]). A quiver $Q$ which is not of finite type is of tame type if and only if the underlying directed graph is a union of Dynkin graphs and extended Dynkin graphs of type $\tilde{A}, \tilde{D}, \tilde{E}$, shown below:

Gabriel proved a stronger statement for quivers of finite type:

Theorem 11 [Gabriel's Theorem, part 2]. The indecomposable representations are in one-to-one correspondence with the positive roots of the corresponding root system. For a Dynkin quiver $Q$, the dimension vectors of indecomposable representations do not depend on the orientation of the arrows in $Q$.

Amazingly, this result is just the tip of an iceberg. Define the Euler form (or Ringel form) of a

In some papers, the definition of tame type includes finite type.

Zigzag Persistence by G. Carlsson and V. de Silva
Zigzag persistence

- Form zigzag module for $X \rightarrow I$
Zigzag persistence

- Form zigzag module for $X \to I$
Zigzag persistence

• Form zigzag module for $X \to I$
Zigzag persistence

- Form zigzag module for $X \to I$
Zigzag persistence

- Form zigzag module for $X \to I$
Persistent homology applied to data

Example: Cyclo-Octane (C₈H₁₆) data
1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Cyclo-Octane (C₈H₁₆) data

1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Persistence intervals in dimension 0:

Persistence intervals in dimension 1:

Persistence intervals in dimension 2:

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Cyclo-Octane (C₈H₁₆) data
1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Cyclo-Octane \((C_8H_{16})\) data

1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Cyclo-Octane \((\text{C}_8\text{H}_{16})\) data
1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Cyclo-Octane (C₈H₁₆) data

1,000,000+ points in 24-dimensional space

Non-Manifold Surface Reconstruction from High Dimensional Point Cloud Data
by Shawn Martin and Jean-Paul Watson, 2010.
Persistent homology applied to data

Example: Equilateral pentagons in the plane

Image credit: Clayton Shonkwiler
Persistent homology applied to data

Image credit: Clayton Shonkwiler
Persistent homology applied to data

- **Stability Theorem.**
  If $X$ and $Y$ are metric spaces, then
  
  $$d_b(\text{PH}(\check{\text{Cech}}(X)), \text{PH}(\check{\text{Cech}}(Y))) \leq 2d_{\text{GH}}(X, Y)$$
Topology applied to image data
The receptive fields of cells in our primary visual cortex (V1) are related to the statistics of natural images.

*Independent component filters of natural images compared with simple cells in primary visual cortex* by JH van Hateren and A van der Schaaf, 1997
Persistent homology applied to data

3x3 high-contrast patches from images
Points in 9-dimensional space, normalized to have average color gray and contrast norm one (on 7-sphere).

Persistent homology applied to data

1. Densest patches according to a global estimate
Persistent homology applied to data

1. Densest patches according to a global estimate

Interpretation: nature prefers linearity
Persistent homology applied to data

2. Densest patches according to an intermediate estimate
Persistent homology applied to data

2. Densest patches according to an intermediate estimate

Interpretation: nature prefers horizontal and vertical directions
Persistent homology applied to data

3. Densest patches according to a local estimate
Persistent homology applied to data

3. Densest patches according to a local estimate
Persistent homology applied to data

3. Densest patches according to a local estimate
Persistent homology applied to data

3. Densest patches according to a local estimate

Interpretation: nature prefers linear and quadratic patches at all angles

Image credit: https://plus.maths.org/content/imaging-maths-inside-klein-bottle
Takens’ Theorem

Roughly speaking: Let $M$ be a $d$-dimensional compact manifold, let $\phi: M \to M$ be a flow, and let $f: M \to \mathbb{R}$ be a measurement. Then generically,

$$m \mapsto (f(m), f(\phi(m), f(\phi^2(m), \ldots, f(\phi^{2d}(m)))$$

is an embedding $M \hookrightarrow \mathbb{R}^{2d+1}$.

Detecting strange attractors in turbulence by Floris Takens, 1982
Conley index theory
Conclusions \ ... so far!

- Datasets have shape, which are reflective of patterns within.
- Persistent homology is a way to measure some of the local geometry and global topology of a dataset.
Why is applied topology popular when few datasets have Klein bottles?

- Many datasets have clusters & flares (as in the diabetes example)
- Motivates interesting questions in many pure disciplines: mathematics, computer science (computational geometry), statistics
- Interest from domain experts in biology, neuroscience, computer vision, dynamical systems, sensor networks, ...
- Materials science, pattern formation
- Machine learning: small features matter
- Agent-based modeling (swarming)

Possible answer: Persistent homology measures both the local geometry and the global topology of a dataset.
Why is applied topology popular when few datasets have Klein bottles?

- Many datasets have clusters & flares (as in the diabetes example)
- Motivates interesting questions in many pure disciplines: mathematics, computer science (computational geometry), statistics
- Interest from domain experts in biology, neuroscience, computer vision, dynamical systems, sensor networks, ...
- Materials science, pattern formation
- Machine learning: small features matter
- Agent-based modeling (swarming)

Possible answer: Persistent homology measures both the local geometry and the global topology of a dataset.
Why is applied topology popular when few datasets have Klein bottles?

- Many datasets have clusters & flares (as in the diabetes example)
- Motivates interesting questions in many pure disciplines: mathematics, computer science (computational geometry), statistics
- Interest from domain experts in biology, neuroscience, computer vision, dynamical systems, sensor networks, ...
- Materials science, pattern formation
- Machine learning: small features matter
- Agent-based modeling (swarming)

Possible answer: Persistent homology measures both the local geometry and the global topology of a dataset.
Topology applied to machine learning:
From global to local

Survey paper joint with Michael Moy

Persistent homology measures both the global topology and the local geometry of a dataset.
Local geometry

Measures of order for nearly hexagonal lattices
Motta, Neville, Shipman, Pearson, Bradley, 2018
Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014
Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skewer, 2014
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models

Topaz, Ziegelmeier, Halverson, 2015
Local geometry

Analysis of Kolmogorov flow and Rayleigh–Bénard convection using persistent homology
Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischakow 2016
Local geometry

Understanding diffusion patterns of glassy, liquid and amorphous materials via persistent homology analysis Onodera, Kohara, Tahara, Masuno, Inoue, Shiga, Hirata, Tsuchiya, Hiraoka, Obayashi, Ohara, Mizuno, Sakata, 2019
Local geometry

Statistical topological data analysis using persistence landscapes
Bubenik, 2015
Perspective images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

\[ \text{Answer: (from left)} \quad r = 1.75, 2, 1.75, 2, 2. \]

Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier; 2017
Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
Local geometry

Hyperbolic disk  Flat disk  Disk on sphere

Persistent homology detects curvature
Bubenik, Hull, Patel, Whittle, 2019
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020
Local geometry

On the choice of weight functions for linear representations of persistence diagrams
Divol and Polonik, 2019
Local geometry

Cellular automata simulation of grain growth.

Jeremy Mason research group @UC Davis
Where can I find resources if I am interested in applied topology?

- You may be interested in the Applied Algebraic Topology Research Network. Become a member to receive email invites to the online research seminars. Recorded talks are available at the YouTube Channel. There is also a forum.
- Another source of applied topology news is appliedtopology.org.
- A second online research seminar is GEOTOP-A: Applications of Geometry and Topology.
- Mailing lists with announcements in applied topology include WinCompTop and ALGTOP-L.

https://www.math.colostate.edu/~adams/advising
Thank you!