CSU Putnam Seminar
Nov 11, 2019

New Problems!

2001 A2 You have coins $C_1, C_2, \ldots, C_n$. For each k, C_k is biased so that, when tossed, it has probability $1/(2k+1)$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n.

1988 A–1 Let R be the region consisting of the points (x,y) of the cartesian plane satisfying both $|x|-|y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.

2009 A1 Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A)+f(B)+f(C)+f(D)=0$. Does it follow that $f(P)=0$ for all points P in the plane?

"Old" Problems

2017 B3 Suppose that $f(x)=\sum_{i=0}^{\infty} c_i x^i$ is a power series for which each coefficient c_i is 0 or 1. Show that if $f(2/3)=3/2$, then $f(1/2)$ must be irrational.

2010 A2 Find all differentiable functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$f'(x)=\frac{f(x+n)-f(x)}{n}$$

for all real numbers x and all positive integers n.

1984 A-2. Express $\displaystyle\sum_{k=1}^{\infty} \frac{6^k}{(3^{k+1}-2^{k+1})(3^k-2^k)}$ as a rational number

2008 A1 Let $f:\mathbb{R}^2\to\mathbb{R}$ be a function such that $f(x,y)+f(y,z)+f(z,x)=0$ for all real numbers x, y, and z. Prove that there exists a function $g:\mathbb{R}\to\mathbb{R}$ such that $f(x,y)=g(x)-g(y)$ for all real numbers x and y.