2001 A2 You have coins $C_1, C_2, \ldots, C_n$. For each $k$, $C_k$ is biased so that, when tossed, it has probability $1/(2k+1)$ of falling heads. If the $n$ coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of $n$.

1988 A1 Let $R$ be the region consisting of the points $(x, y)$ of the cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region $R$ and find its area.

2009 A1 Let $f$ be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points $P$ in the plane?

2017 B3 Suppose that $f(x) = \sum_{j=0}^{\infty} c_j x^j$ is a power series for which each coefficient $c_j$ is 0 or 1. Show that if $f(2/3) = 3/2$, then $f(1/2)$ must be irrational.

2010 A2 Find all differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that
$$f'(x) = \frac{f(x+n)-f(x)}{n}$$
for all real numbers $x$ and all positive integers $n$.

1984 A2. Express $\sum_{k=1}^{\infty} \frac{b^k}{(3^k+1)(3^k-2^k)}$ as a rational number.

2008 A1 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $f(x, y) + f(y, z) + f(z, x) = 0$ for all real numbers $x, y, z$. Prove that there exists a function $g : \mathbb{R} \to \mathbb{R}$ such that $f(x, y) = g(x) - g(y)$ for all real numbers $x$ and $y$. 