CSU Putam Club
Oct 28, 2019

"Old" Problems

(2017 B3) Suppose $f(x) = \sum_{i=0}^{\infty} c_i x^i$ is a power series such that each coefficient c_i is either 0 or 1. Show that, if $f(2/3) = 3/2$, then $f(1/2)$ cannot be rational.

(2002 A1) Let k be a fixed positive integer. The nth derivative of $\frac{1}{x^k-1}$ has the form

$$\frac{P_n(x)}{(x^k-1)^{n+1}}$$

where $P_n(x)$ is a polynomial. Find $P_n(1)$.

2010 A2 Find all differentiable functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f'(x) = \frac{f(x+n) - f(x)}{n}$$

for all real numbers x and all positive integers n.

1984 A-2. Express $\displaystyle\sum_{k=1}^{\infty} \frac{6^k}{(3^{k+1}-2^{k+1})(3^k-2^k)}$ as a rational number

2008 A1 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $f(x,y) + f(y,z) + f(z,x) = 0$ for all real numbers x, y, and z. Prove that there exists a function $g : \mathbb{R} \to \mathbb{R}$ such that $f(x,y) = g(x) - g(y)$ for all real numbers x and y.

New Problems!

2001 A1 Consider a set S and a binary operation $*$, i.e., for each $a, b \in S$, $a * b \in S$. Assume $(a * b) * a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$.

2001 A2 You have coins $C_1, C_2, \ldots, C_n$. For each k, C_k is biased so that, when tossed, it has probability $1/(2k+1)$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n.