Sept 25

Note Title 9/25/2009

Differentiation

Approximation

$f : D \to \mathbb{R}$

$D \subseteq \mathbb{R}$

$a \in D^{int}$

Approx. f near a.
by polynomials

0'th deg poly.
$c(x) = c$.

What is the best choice for c to approx. f near a?

We'll $f(a)$.

Is this a good choice?

$$\lim_{x \to a} |\underline{f}(x) - \underline{c}(x)|$$

$$= \lim_{x \to a} |f(x) - f(a)|$$

If f is cont. then this $= 0$ + is a reasonable approx.

1st order approx.

$$L(x) = c + \boxed{m(x-a)}$$

$$c = f(a)$$

$$= f(a) + m(x-a)$$

$$\lim_{x \to a} \frac{|f(x) - L(x)|}{|x-a|} = 0$$

in order for the approx to be good.

$$\left| \frac{f(x) - f(a) - m(x-a)}{x-a} \right|$$

$$= \left| \frac{f(x) - f(a)}{x-a} - m \right|$$

Just saying

$$\lim_{x \to a} \frac{f(x) - f(a)}{x-a} = \underline{\underline{m}}.$$

$$\shortparallel$$

$$f'(a)$$

---

1) If $a \in D^{int}$, f has a max or min at a & f is diff at a then

$$\underline{f'(a) = 0}$$

2) Rolle's Thm.

If f is cont. on $[a,b]$, diff. on (a,b) & $f(a) = f(b)$

then $\exists\, c \in (a,b)$
with $f'(c) = 0$.

<u>Thm</u> (Generalized Mean Value Thm)

Suppose f, g are cont on $[a,b]$ + diff. on (a,b).

+ $g'(x) \neq 0$ on (a,b)

Then $\exists\, c \in (a,b)$

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

<u>pf</u>

$$h(x) = (g(x)-g(a))(f(b)-f(a)) - (f(x)-f(a))(g(b)-g(a))$$

$h(a) = 0$ //
$h(b) = 0$ //

h - cont on $[a,b]$
+ diff on (a,b).

$\exists\, c \in (a,b),\ h'(c) = 0$

$$h'(x) = g'(x)(f(b)-f(a)) - f'(x)(g(b)-g(a))$$

$$g'(c)(f(b)-f(a)) = f'(c)(g(b)-g(a))$$

$g(b) \neq g(a)$ by Rolle's Thm

(if $= \Rightarrow \exists c, g'(c) = 0$)

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$f : \mathbb{R} \to \mathbb{R}^n$.

$f = (f_1, f_2, \ldots, f_n)$

To say f is diff. at a should mean f has a good 1st order approx.

$$L(x) = (f_1(a) + m_1(x-a), f_2(a) + m_2(x-a), \dots, f_n(a) + m_n(x-a))$$

$$\lim_{x \to a} \frac{\| f(x) - L(x) \|}{|x-a|} = 0$$

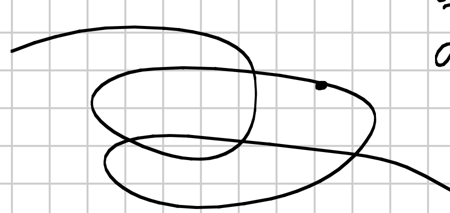
$\iff$ $i = 1, \dots, n$

$$\lim_{x \to a} \left| \frac{f_i(x) - f_i(a)}{x-a} - m_i \right| = 0.$$

$\iff$ each f_i is diff. //

Such fc'ns are curves in $\mathbb{R}^n$

derivative is a vector $(f_1', f_2', \dots, f_n')$.



$$L(x) = f(a) + f'(\vec{a})\underline{\underline{(x-a)}}$$

$$f : \mathbb{R}^n \to \mathbb{R}$$

1st order polynomials
in n-variables

$$L(\vec{x}) = f(\vec{a}) + c_1(x_1 - a_1) + c_2(x_2 - a_2) \cdots c_n(x_n - a_n)$$

$$= f(\vec{a}) + \underbrace{\vec{c} \cdot (\vec{x} - \vec{a})}_{\text{linear part}}$$

& I want

$$\lim_{\vec{x} \to \vec{a}} \frac{|f(\vec{x}) - L(\vec{x})|}{\|\vec{x} - \vec{a}\|} = 0$$

Computing c_i

For c_1,
Look along the $\vec{e}_1$-dir.

$$\lim_{h \to 0} \left| \frac{f(\vec{a} + h\vec{e}_1) - f(\vec{a})}{h} - c_1 \right|$$

$$f(\vec{a}+h\vec{e}_1) - f(a)$$
$$= f(a_1+h, a_2, a_3, \cdots, a_m)$$
$$- f(a_1, a_2, a_3 \cdots a_n)$$

+ So this limit being 0 means just that

$$c_1 = \frac{\partial f}{\partial x_1}(\vec{a})$$

$$c_i = \frac{\partial f}{\partial x_i}(\vec{a})$$

$$L(x) = f(\vec{a}) + \frac{\partial f}{\partial x_1}(\vec{a})(x_1 - a_1) + \cdots + \frac{\partial f}{\partial x_n}(\vec{a})(x_n - a_m)$$

$\partial_{x_i}(f)$, $\partial_i(f)$, f_i.