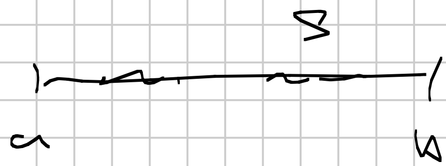
Sept 21

Note Title 9/21/2009

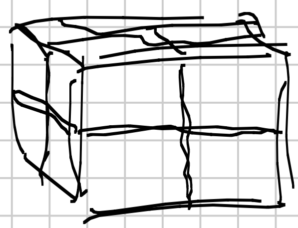
#8



HW
p 33, 38, 41
#4

in $\mathbb{R}^n$

Boxes



$[a_i, b_i]$ - bounds
i'th coord. of S

- one approach -

2nd approach -

S - inf.

1) $x_1 \in S$ as $S \neq \emptyset$

2) $x_2 \in S - \{x_1\} \neq \emptyset$

$x_1 \neq x_2$

3) $x_3 \in S - \{x_1, x_2\} \neq \emptyset$

$x_1 \neq x_2 \neq x_3$

$\vdots$

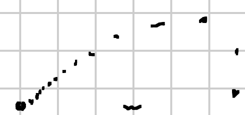
$k+1$) $x_{k+1} \in S - \{x_1, \dots, x_k\} \neq \emptyset$

all x_i - diff.

$\{x_i\}$ - bounded.

$\exists\ x_{k_i} \to \vec{a}$.

$\bar{a}$ is an accum. pt. of S.



If some $x_{k_i} = \vec{a}$

take

$$y_j = x_{k_{i+j}}$$

all $y_j \in S$, $y_j \to \vec{a}$

No $y_j = \vec{a}$.

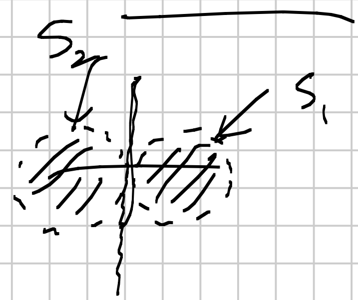
Connected Sets

Def For $S \subseteq \mathbb{R}^n$ we say S is disconnected if $\exists$ 2 sets S_1, S_2 so that

1) $S_1, S_2 \neq \emptyset$

2) $S = S_1 \cup S_2$

3) $\overline{S_1} \cap S_2 = S_1 \cap \overline{S_2} = \emptyset$



If S is not disconnected then we say it is connected.

Thm A set $S \subseteq \mathbb{R}$ is connected iff it is an interval.

pf Suppose S is not an int.,

$\Rightarrow \exists$ 3pts $a < b < c$, $a, c \in S$, $b \notin S$.

$S_1 = (-\infty, b) \cap S$

$S_2 = (b, \infty) \cap S$

1) $S_1, S_2 \neq \emptyset$

2) $S_1 \cup S_2 = S$

3) $\overline{S_1} \cap S_2 \subseteq \overline{(-\infty, b)} \cap (b, \infty) = \emptyset$

$$S_1 \cap \overline{S_2} \subseteq (-\infty, b) \cap \overline{(b, \infty)}$$
$$= \emptyset$$

$\longrightarrow$

Suppose S is an interval but is disconnected.

i.e. $\exists\ S_1, S_2 \neq \emptyset$

$S = S_1 \cup S_2$

& $\overline{S_1} \cap S_2 = S_1 \cap \overline{S_2} = \emptyset$

Let $a \in S_1$

$b \in S_2$

assume $a < b$.

Look at $[a,b] \subseteq S$

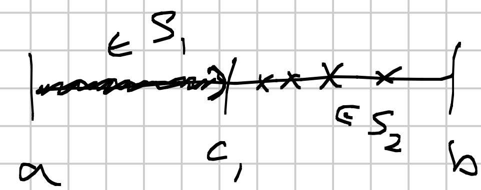
as S is an int.

Let $c_1 = \sup(\{c \in [a,b] : [a,c] \subseteq S_1\})$

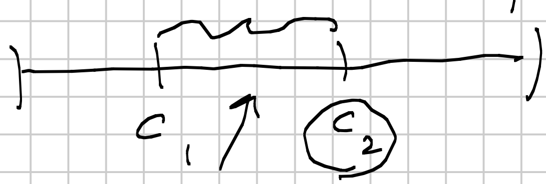
Let $c_2 = \inf(\{c \in [a,b] : c \in S_2\})$

Claim $c_1 = c_2$

1) $c_1 \leq c_2$



2) Suppose gap



No pts. in S_2

<u>all</u> in S_1 $\Rightarrow$ c_1 too small

<u>No gap</u>

$$\Rightarrow c \begin{cases} c_1 \in \overline{S_1} \\ \quad \| \\ c_2 \in \overline{S_2} \end{cases}$$

$c \in [a, b]$, $c \in S$.

if $c \in S_1 \Rightarrow c \in S_1 \cap \overline{S_2} = \emptyset$

$c \in S_2 \Rightarrow c \in S_2 \cap \overline{S_1} = \emptyset$

$\Rightarrow\Leftarrow$

So S is conn.

Thm If $C \subseteq \mathbb{R}^n$ is connected + $f: C \to \mathbb{R}^m$ is cont.

Then $f(C)$ is connected.

pf If $S \subseteq \mathbb{R}^m$ is disc., f is cont + onto S.

Then $f^{-1}(S)$ is disc.

$$S = S_1 \cup S_2$$
$$S_1, S_2 \neq \emptyset$$
$$\overline{S_1} \cap S_2 = S_1 \cap \overline{S_2} = \emptyset$$

$$f^{-1}(S) = f^{-1}(S_1) \cup f^{-1}(S_2)$$

+ as f is onto

$$f^{-1}(S_1), f^{-1}(S_2) \neq \emptyset$$

$f^{-1}(\overline{S_1})$ - closed.

& contains $f^{-1}(S_1)$

so $f^{-1}(\overline{S_1}) \supseteq \overline{f^{-1}(S_1)}$

$$f^{-1}(\overline{S_1}) \cap f^{-1}(S_2)$$

$$= f^{-1}(\overline{S_1} \cap S_2) = \emptyset$$

$$\Rightarrow \overline{f^{-1}(S_1)} \cap f^{-1}(S_2) = \emptyset$$

$$\& \quad f^{-1}(S_1) \cap \overline{f^{-1}(S_2)} = \emptyset.$$

& so

$f^{-1}(S)$ is disc.

Thm Suppose

$S \subseteq \mathbb{R}^n$ is conn.

$f : S \to \mathbb{R}$ is cont.

$\&$ $\exists$ $a < b \in \mathbb{R}$,

with pts. $\vec{x}, \vec{y} \in S$

$f(\vec{x}) = a$, $f(\vec{y}) = b$

Then for every

$a < c < b$

$\exists$ $\vec{z} \in S$, $f(\vec{z}) = c$.

<u>pf</u> S - connected

& f is cont.

so $f(S) \subseteq \mathbb{R}$

is connected.

$\Rightarrow$ $f(S)$ is an

interval.