Oct 7

Note Title 10/7/2009

1) Foundations of $\mathbb{R}^n$.

dot products

C. Schwartz inequ

Δ-inequ

Open sets

Closed sets

interior

closure

boundary

"C"'s

Completeness

Compactness

Connectedness

Convexity

Cauchy'ness.

Uniform Continuity

Differentiation

Foundations

1 var Calculus

C^1- fctns.

Chain Rule

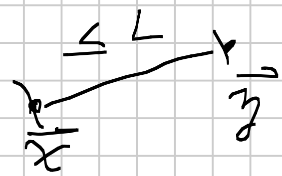
#2) For $S \subseteq \mathbb{R}^n$ to be bounded means $\exists\, B > 0$ and for all $\vec{x} \in S$, $\|\vec{x}\| < B$.

$\operatorname{dia}(S) < \infty$

$\;\;\, \parallel$

$\;\;\, L \implies \forall\, \vec{x}, \vec{y} \in S,$

$\|\vec{x} - \vec{y}\| \leq L$



Case 1: $S = \emptyset$

$B = \cancel{47}\ 1$

$\forall \vec{x} \in S,\ \|\vec{x}\| < 47$

because there are <u>NO</u> x's.

<u>Case</u> 2: $S \neq \emptyset$

Let $\vec{x}_0 \in S$.

Set $B = \|\vec{x}_0\| + L$.

For $\vec{x} \in S$

$$\|\vec{x}\| = \|\vec{x} - \vec{x}_0 + \vec{x}_0\|$$
$$\leq \|\vec{x} - \vec{x}_0\| + \|\vec{x}_0\|$$
$$\leq L + \|\vec{x}_0\| = B.$$

b) $K \subseteq \mathbb{R}^n$ compact & not $\emptyset$.

K - compact -

1) Every open cover has a finite subcover

- or -

2) Closed & bounded

– or –

3) Every sequence has a conv. subsegence that conv. to a point of K.

For $\varepsilon = \frac{1}{n}$, there exist pts $x_n, y_n \in K$

& $\|x_n - y_n\| \geq \operatorname{dia}(K) - \frac{1}{n}$.

$x_{n_i} \to x_0$

y_{n_i} – Take a subsequence

$y_{n_{i_k}} \to y_0$, $x_{n_{i_k}} \to x_0$

$\|x_0 - y_0\| = \lim_{k \to \infty} \|x_{n_{i_k}} - y_{n_{i_k}}\|$

$$\geq \operatorname{dia}(K) - \lim_{k\to\infty} \frac{1}{n_{ik}}$$

$$= \operatorname{dia}(K)$$

$$K \subseteq \mathbb{R}^n$$

$$(K \times K) \subseteq \mathbb{R}^{2n}$$

$$= \{(x, y) : x, y \in K\}.$$

If K is compact then $K \times K$ is compact.

$$f(\vec{x}, \vec{y}) = \|\vec{x} - \vec{y}\|$$

Continuous on $K \times K$

It achieves its sup which is $\operatorname{dia}(K)$.

3) $2x^2 + y^2 + z^2 = 4$

$$(t^2, t, t)$$

$$2t^4 + 2t^2 = 4$$

$$t^4 + t^2 = 2$$

$$t^4 + t^2 - 2 = 0$$

$$\underline{\underline{t^2}} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$= 1, \cancel{-2}$$

$$t = \pm 1$$

pts $(1, 1, 1)$

$(1, -1, -1)$

Show at these pts
∇f is $\parallel$ to tan to curve.

$$\nabla(2x^2 + y^2 + z^2)$$

$$(4x, 2y, 2z)$$

$$(t^2, t, t)' = (2t, 1, 1)$$

2 pts
$(1, 1, 1)$

$$\nabla f = (4, 2, 2)$$

tan. = (2, 1, 1) - parallel

$\overline{(1, -1, -1)}$

$\nabla(f) = (4, -2, -2)$

tan = (2, -1, -1) - parallel