Nov 30

Note Title 11/30/2009

HW
p158 # 6, 8a + c
p167 #2

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85 A

75

65 B

50 C

↓

35

$F(s,t) = (s+t, t^2, t^3)$

1) F is 1-1.

If $F(s,t) = F(s_1, t_1)$
then $s = s_1$ + $t = t_1$

$s + t = s_1 + t_1$
$t^2 = t_1^2$
$t^3 = t_1^3$.

x^3 is st. inc.
so $t = t_1$
$\Rightarrow$ $s = s_1$.

Where is Σ - near
$(a, b, c) = F(s_0, t_0)$
a smooth sfc.?

Look at DF
3 X 2 matrix.

Σ is smooth wherever
DF has rank 2.

$$\begin{bmatrix} 1 & 1 \\ 0 & 2t \\ 0 & 3t^2 \end{bmatrix}$$ ~ Rank 2
wherever $t \neq 0$

Another way -
Σ is smooth at
(a, b, c) if one of
var. can be solved for

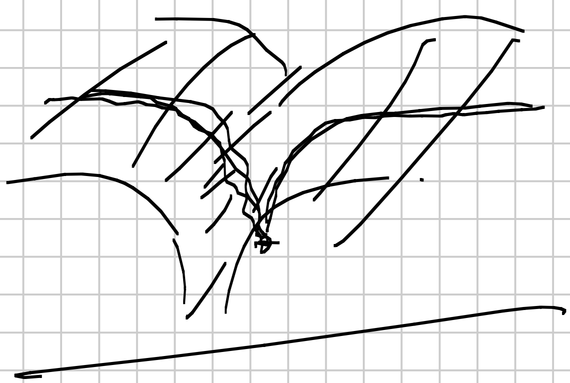
as a C^1-fctn of the other two.

$$\overline{(a, b, c)} = (s_0 + t_0,\ t_0^2,\ t_0^3$$

$$t_0 = c^{1/3}$$

$$b = c^{2/3} = f(a, c)$$

$$\frac{\partial f}{\partial c} = \frac{2}{3c^{1/3}}$$. - undef. if $\underline{\underline{c = 0}}$.



$$\vec{n} = \begin{bmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 2t & 3t^2 \end{bmatrix}$$

$$= (0, \underline{\underline{-3t^2, 2t}})$$

$$\lim_{x \to 0} \left(\frac{\sin x - x\cos x - cx^3}{x^5} \right)$$

$$\lim_{x \to 0} \frac{R_5(x)}{x^5} = 0$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{5!} + R_5^s(x)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + R_5^c(x)$$

$$\frac{\cancel{x} - \frac{x^3}{6} + \frac{x^5}{5!} - \cancel{x} + \frac{x^3}{2} - \frac{x^5}{4!} - cx^3 + R_5^s(x) - xR_5^c(x)}{x^5}$$

$$\frac{x^3\left(\frac{1}{2} - \frac{1}{6} - c\right)}{x^5} + \frac{\left(\frac{1}{4!} - \frac{1}{5!}\right)\cancel{x^5}}{\cancel{x^5}} + \frac{R_5^s(x)}{x^5} - \frac{R_5^c(x)}{x^4}$$

$$\to 0 \qquad \to 0$$

$$= -\frac{1}{30}$$

$$\frac{\left(\frac{1}{3} - c\right)}{x^2}$$

For lim to exist $c = 1/3$

Integration

Def. $S \subseteq \mathbb{R}$ has "zero content" if for all $\varepsilon > 0$ $\exists$ a finite set of intervals $I_1, I_2, \ldots I_k$ with

1) $S \subseteq \bigcup_{i=1}^{k} I_k$ &

2) $\sum_{i=1}^{k} \ell(I_k) < \varepsilon$

Saw if $f : [a,b]$ is bded & its set of discont. has 0-content then f is integrable.

Defn $S \subseteq \mathbb{R}$ has "measure 0" if for all $\varepsilon > 0$ $\exists$ a countable list of intervals

$I_1, I_2, \ldots$

with

1) $S \subseteq \bigcup_{i=1}^{\infty} I_i$

2) $\sum_{i=1}^{\infty} \ell(I_i) < \varepsilon$.

Ex $S = \mathbb{Q} \cap [0,1]$

$S \subseteq \bigcup_{i=1}^{k} I_i$, I_i - closed

$\bigcup_{i=1}^{k} I_i$ is a closed set.

it contains $\overline{S}$,

so $\bigcup_{i=1}^{k} I_i \supseteq [0,1]$.

+ $\sum_{i=1}^{k} \ell(I_i) \geq 1$,

$S = \{q_1, q_2, q_3 \ldots\}$

$$I_k = \left(q_k - \frac{\varepsilon}{2\cdot 2^k},\ q_k + \frac{\varepsilon}{2\cdot 2^k}\right)$$

Count. list.

$$\ell(I_k) = \frac{\varepsilon}{2^k}$$

$$\sum_{k=1}^{\infty} \ell(I_k) = \varepsilon \sum_{k=1}^{\infty} \frac{1}{2^k} = \varepsilon.$$

$$S \subseteq \bigcup_{k=1}^{\infty} I_k.$$

<u>Thm</u> $f : [a,b] \to \mathbb{R}$ is bded.
f is R. int. iff the disc. of f form a set of measure 0.
(Henri Lebesgue)

<u>Thm</u> If f is int on $[a,b]$ so is f^2.

pf. $\operatorname{disc}(f^2) = \operatorname{disc}(f)$

$\overline{(f \cdot g)}$

$(f+g)^2 = f^2 + 2fg + g^2$

int int

so, if $f+g$

are int so is fg.