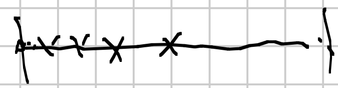
Dec 7

Note Title 12/7/2009

x_i - conv. sequ.

$S = \{x_i\}$ - show this has <u>0-content</u>

$x_i = \frac{1}{i}$



$\varepsilon > 0$ /

$\left(-\frac{\varepsilon}{8}, \frac{\varepsilon}{8}\right)$ - covers a lot.

Not covered if

$$\frac{1}{i} \geq \frac{\varepsilon}{8}$$

$1 \leq i \leq \frac{8}{\varepsilon}$ - only finitely <u>many</u>

Say K of them

For $1 \leq i \leq \frac{8}{\varepsilon}$, Take $\left(\frac{1}{i} - \frac{\varepsilon}{8K}, \frac{1}{i} + \frac{\varepsilon}{8K}\right)$

$\overline{K}$ - intervals

sum of lengths $< \varepsilon/4$

General case

Let $x_k \to L$

Let $\varepsilon > 0$ -

Let $I_0 = \left(L - \frac{\varepsilon}{8}, L + \frac{\varepsilon}{8}\right)$.

b/s $x_k \to L$,

$\exists K$, $k \geq K$,

$|x_k - L| < \varepsilon/8$.

Let I_i, $i = 1 \cdots K-1$

be $\left(x_i - \frac{\varepsilon}{8K}, x_i + \frac{\varepsilon}{8K}\right)$

$I_0, I_1 \cdots I_{K-1}$.

Given x_i,

$i \geq K \Rightarrow x_i \in I_0$.

$i < K \Rightarrow x_i \in I_i$.

$\{x_i\} \subseteq \bigcup_{i=0}^{K-1} I_i$.

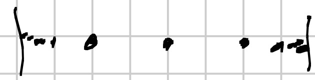
$$\sum_{i=0}^{K-1} \ell(I_i)$$

$$= \frac{\varepsilon}{4} + (K-1)\left(\frac{\varepsilon}{4K}\right)$$

$$< \varepsilon.$$

Suppose
$S \subseteq \mathbb{R}$ is
bounded
+ the set of accumulation pts of S has
0-content,
Show S has 0-content.

Ex $\quad x_i = \begin{cases} \frac{1}{i}, & i\text{-even} \\ 1-\frac{1}{i}, & i\text{-odd.} \end{cases}$



Step 1.

Select open intervals $I_1 \dots I_k$, cover accum pts of S & $\sum_{i=1}^{k} \ell(I_i) < \varepsilon/2$.

Claim $S - \bigcup_{i=1}^{k} I_i$ has no accum pts.

if it did, then $\exists\; x_i \in S - \bigcup_{i=1}^{k} I_i$

$x_i \to a$ - accum pt.

$\Rightarrow$ a is an accum pt of S.

$\Rightarrow a \in I_k$, I_k open

$\Rightarrow$ some $x_i \in J_k$.

We assume $x_i \notin \bigcup_k J_k$

$$\Rightarrow \overline{S - \bigcup_{i=1}^{K} J_i} \text{ is finite}$$

$$= \{x_1, x_2, \dots, x_L\}.$$

$$I_{K+j} = \left(x_j - \frac{\varepsilon}{8L}, x_j + \frac{\varepsilon}{8L}\right)$$

$$\overline{\& \quad S \subseteq \bigcup_{i=1}^{K+L} I_i}$$

$$\sum_{i=1}^{K+L} \ell(I_i) < \frac{3}{4}\varepsilon$$