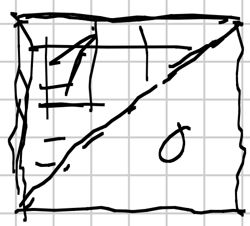
Dec 11

Note Title 12/11/2009

$$g(x,y) = \begin{cases} 1, & x \le y \\ 0 & \text{otherwise} \end{cases}$$

$$[0,1) \times \underline{[0,1]}.$$



$$S = \{(x,y) : x \le y,\ x,y \in [0,1]\}$$

I want to show
S is measurable

$\iff$ ∂S has 0-content

$$\partial S = \{(x,1) : x \in [0,1]\} \cup \{(0,y) : y \in [0,1]\} \cup \{(x,x) : x \in [0,1]\}$$

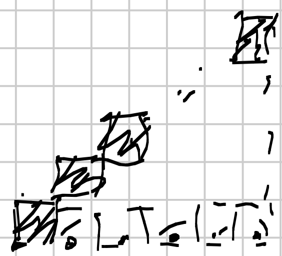
Given $\varepsilon > 0$,

$\varepsilon/3$ for each.

$$R_1 = [0,1] \times [-\varepsilon/6, 1+\varepsilon/6]$$

$$R_2 = [-\varepsilon/6, \varepsilon/6] \times [0,1]$$

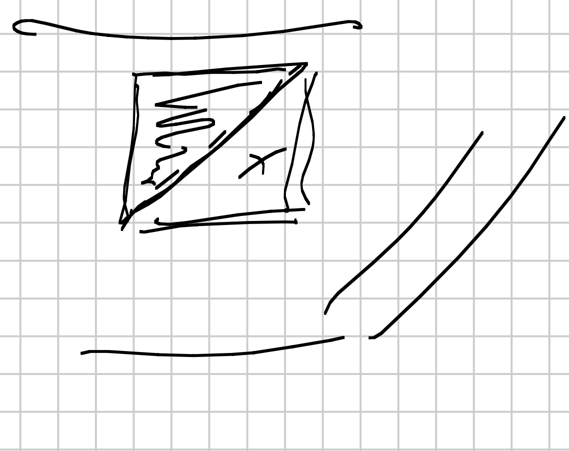
R_2 = 



Choose n, $\frac{1}{n} < \varepsilon/3$

$R_3, \ldots, R_{n+2}$ $\quad \left[\frac{k}{n}, \frac{k+1}{n}\right] \times \left[\frac{k}{n}, \frac{k+1}{n}\right]$

$$k = 0, 1, 2 \ldots n-1$$



$S = \{(x,y) : x, y \in \mathbb{Q}, \ x, y \in [0,1]\}$.

$S \subseteq [0,1] \times [0,1]$ &

so is bounded

$\partial S = [0,1] \times [0,1]$.

Take $(x,y) \in [0,1] \times [0,1]$.

$\exists$ rat'ls $p_i \to x$, $p_i \in [0,1]$

& $q_i \to y$, $q_i \in [0,1]$

so $(p_i, q_i) \in S$

& $(p_i, q_i) \to (x,y)$

$\exists$ irrat's $u_i \to x$, $u_i \in [0,1]$

$v_i \to y$ $v_i \in [0,1]$

so $(u_i, v_i) \in S^c$

& $(u_i, v_i) \to (x,y)$

$\Rightarrow (x,y) \in \partial S$.

& Can't be bigger

as ∂S is closed

+ in $S^c \subseteq [0,1]\times[0,1]$

Cannot cover
$[0,1]\times[0,1]$ by
a finite union of
rect. whose areas
add to < 1

Suppose
$C = [0,1]\times[0,1]$ is
covered by
$R_1 \cup R_2 \ldots R_k$ -
rectangles +
$\sum_{i=1}^{k} Area(R_i) < 1.$

$f = \chi_C, \quad g = \sum_{i=1}^{k} \chi_{R_i}$

$0 \leq f \leq g$

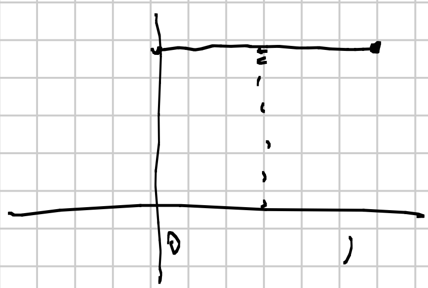
$\int_R f \leq \int_R g$

$$1 \le \sum_{i=1}^{k} \int \chi_{R_i}\, dA$$

$$\sum_{i=1}^{k} \text{Area}(R_i) < 1$$

$$\Rightarrow \Leftarrow$$

S is not J. msble



$$\int_0^1 e^{-x^2}\, dx$$

Max(e^{-x^2}) on S is 1

on S
$$0 \le e^{-x^2} \le 1$$

SO ON S

$$0 \le e^{-x^2} \le \chi_S$$

$$0 \le \iint_S e^{-x^2}\, dA \le \iint \chi_S \, dA = 0$$