HW 9
Math 261

Please see the course syllabus for details on how to turn in your homework assignments.

1. (5 pts.) TRUE OR FALSE:
   (a) If $f(x, y, z)$ is a differentiable function, then the gradient of f is a conservative vector field.
   (b) If $\mathbf{F}$ is a conservative vector field on a region R and C is a closed curve bounding R then the line integral of $\mathbf{F}$ along C equals 0.
   (c) If $\mathbf{F}$ is a conservative vector field on $\mathbb{R}^3$ and C_1, C_2 two paths starting at a point P and ending at a point Q then the line integral of $\mathbf{F}$ along C_1 equals the line integral of $\mathbf{F}$ along C_2.
   (d) If $\mathbf{F}$ is a conservative vector field, its potential function is unique.
   (e) If $\mathbf{F}$ is a conservative vector field, its potential function is unique up to translation by a constant.
2. (3pts.) Give an example of a conservative vector field, and of a vector field which is not conservative.
3. (3 pts.) Suppose conservative vector field $\mathbf{G}$ has potential function $g(x, y, z) = x^2 + yz$. Compute the work done when moving through this vector field along any simple curve from from $(0, 1, 1)$ to $(2, 0, 1)$.
4. (3 pts.) Find *the* potential function $f(x, y, z)$ for vector field

$$\mathbf{F} = \langle \sin(y), x\cos(y) + z\cos(y), \sin(y) + 2z\rangle$$

   such that $f(9, 0, 1) = 2$. You may assume that $\mathbf{F}$ is conservative.
5. (3 pts.) Use the component test ($M_y = N_x$, etc.) to show that the vector field

$$\mathbf{H} = \langle ze^{xz} - \sin(x + 2y), \frac{1}{y} - 2\sin(x + 2y) + 1, xe^{xz} + \frac{1}{z}\rangle$$

   is conservative. (Your solution should consist of three equalities.)
6. (3 pts.) Consider the vector field $\mathbf{F} = \langle y, -x, 0\rangle$. Describe (or even better, sketch) how the vector field looks like for points of the unit circle in the xy- plane. Argue, without doing the component test, that the vector field cannot be conservative.