

HW 7
Math 261

Please see the course syllabus for details on how to turn in your homework assignments.

1. (5 pts.) TRUE OR FALSE:

- (a) Let R denote a region in the xy -plane, given polar coordinates (r, θ) . The expression $\int_R drd\theta$ computes the area of R .
- (b) Let R denote a region in the xy -plane, given polar coordinates (r, θ) . The expression $\int_R r drd\theta$ computes the area of R .
- (c) Let R denote the plane region bounded by a circle of radius one centered at the origin. Then $\int_R x dx dy = 0$.
- (d) Let R denote the plane region bounded by a circle of radius one centered at the origin and (r, θ) polar coordinates. Then $\int_R r drd\theta = 0$.
- (e) If (x, y) are cartesian coordinates and (r, θ) polar coordinates for the plane, and R some plane region, the expression

$$\int_R f(x, y) dx dy = \int_R g(r, \theta) dr d\theta$$

is true when

$$g(r, \theta) = r f(r \cos(\theta), r \sin(\theta)).$$

- 2. (3 pts.) Set up but do **NOT** evaluate a double integral to compute the integral of $f(x, y) = \cos(xy)$ over the part of the unit disk (the region inside the circle of radius 1 centered at the origin) in the first quadrant (where $x > 0, y > 0$).
- 3. (3 pts.) Convert the following double integral to an equivalent polar form but do **NOT** evaluate:

$$\int_0^1 \int_y^{\sqrt{4-y^2}} x^2 + y^2 dx dy$$

- 4. (3 pts.) Set up but do **NOT** evaluate a triple integral to compute the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 1)$. The top plane of the tetrahedron is given by $2x + y + 2z = 2$. **USE** the order $dz dy dx$.
- 5. (3 pts.) Consider the tetrahedron T with vertices $(1, 0, 0)$, $(1, -1, 1)$, $(1, 1, 1)$, and $(0, 0, 1)$. How many regions must T be split into in order to integrate some function over T with the following variable orders (each worth 1 point)? (Each answer is just 1 number!)

- (a) $dx dy dz$
- (b) $dx dz dy$
- (c) $dy dz dx$

(It would be good practice to try setting these integrals up, but that's not required for the problem.)

- 6. (3 pts.) Let R be the region colored in black in the figure below. The two curves bounding R are the circle $x^2 + y^2 = 1$ and the curve described in polar coordinates by the equation $r = 2 \sin(2\theta)$. Set up but do **NOT** evaluate a (sum of) double integral(s) in *polar* coordinates to find the area of R .

