

Math 261 Exam 2 Review Formulas & Reminders

Dan Bates, Spring 2015

Here are a few formulas that might be handy for Exam 2. You *cannot* bring this to the exam, but hopefully it helps with studying....

WARNING: I do not guarantee that this is a comprehensive list! Also, please note that there are various alternative formulations for some of these formulas – I am just picking those that I like the best. Finally, there could be typos – beware!

- Basic limit computations
- Know how to find two paths with different limits to show that a limit doesn't exist (for functions with 2 or more variables).
- Multivariate chain rule: Draw the diagram of dependencies and trace from the function at the top to the desired variable at the bottom in each possible way.
- The derivative of the function f in the direction $u = \langle u_1, u_2, u_3 \rangle$ at the point $p = (p_1, p_2, p_3)$ is $D_u f(P) = \nabla f(P) \cdot \frac{u}{|u|}$. The direction of greatest increase is $u = \nabla f(P)$ – be sure to divide by the magnitude if a unit vector is needed.
- The equation for the plane tangent to the level surface $f(x, y, z) = a$ at point $p = (x_0, y_0, z_0)$ is $f_x(p)(x - x_0) + f_y(p)(y - y_0) + f_z(p)(z - z_0) = 0$. Similarly, the plane tangent to the graph of $z = f(x, y)$ at $p = (x_0, y_0)$ has equation $0 = f_x(p)(x - x_0) + f_y(p)(y - y_0) - (z - f(p))$.
- The linearization of $f(x, y)$ at $p = (x_0, y_0)$ is $L(x, y) = f(p) + f_x(p)(x - x_0) + f_y(p)(y - y_0)$.
- The error in linearizing at p over the region R is bounded as follows: $|E(x, y)| \leq \frac{M}{2}(|x - x_0| + |y - y_0|)^2$, where M is an upper bound on the absolute value of all second derivatives over the region R .
- To find the critical points of $f(x, y)$, set all first partials to 0 (i.e., $f_x = f_y = 0$) and find all solutions. Points of discontinuity are also critical points. For each critical point, compute $f_{xx}f_{yy} - f_{xy}^2$. If less than zero, this is a saddle point. If greater than 0, it is a local min (if $f_{xx} > 0$) or a local max (if $f_{xx} < 0$). If $f_{xx}f_{yy} - f_{xy}^2 = 0$, then the test is inconclusive.
- To find the absolute max and min over a bounded region R (given by inequality constraints), first find all critical points in the plane, discarding any not in R . Then find all critical points along each portion of the boundary (being sure to include all intersections between the various portions of the boundary, e.g., the points of a rectangle). Finally, for all points of interest, compute the function value and choose the max and the min.
- For Lagrange multipliers (problems with equality constraints), choose the objective function f and the constraint g . (f is sometimes $x^2 + y^2$ when you are seeking the point nearest the origin.) Solve the system of equations

$$\nabla f = \lambda \nabla g$$

$$g = 0$$

for x , y , and λ . The max and min will be among these solutions. (Solving the system can be tricky. The best thing you can do to prepare is to try lots of problems.)