HW 5
Math 261, S19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on **Friday, March 8**.

1. (3 pts.) TRUE OR FALSE:
   (a) If $f(x, y, z)$ is a differentiable function and P is a point of maximum in the interior of the domain of f, then the gradient of f at P is the $\mathbf{0}$ vector.
   (b) If $f(x, y, z)$ is a differentiable function and the gradient of f at P is the $\mathbf{0}$ vector, then P is a point of maximum.
   (c) If $f(x, y, z) = 3x + 4y + 5z$ and R is a plane region bounded by a polygon, then the maximum of f on R is obtained at a vertex of the polygon.
2. (3 pts.) Find the derivative of $h(x, y, z) = x + 2y^2 + 3z^3$ at the point $(2, 0, \sqrt{2})$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 0 \rangle$.
3. (3 pts.) Find the equation for the tangent plane to the surface $x^2 - xy - y^2 - z = 0$ at the point $(1, 1, -1)$. Please give your answer in the form $Ax + By + Cz = D$.
4. (3 pts.) Give the best possible upper bound (using the technique from class, i.e. using a linear approximation) for the error in approximating $f(x, y) = x^2 + 3xy - 2y^2$ at the point $(1, 1)$, over the rectangle $|x - 1| \leq 0.1$, $|y - 1| \leq 0.3$. It is OK to leave your answer as a numerical expression (i.e., not simplified down to a number).
5. (3 pts.) Let f be some function of the plane, such that $(1, 1)$ and $(1, -1)$ are critical points. Suppose $f_{xx} = x + 2$, $f_{xy} = x + y - 2$, and $f_{yy} = y + 1$. Classify (min/max/SP) the critical points $(1, 1)$ and $(1, -1)$, clearly indicating any computed values you used to make your decision.
6. (5 pts.)
   Consider the sphere S defined by the equation $x^2 + y^2 + (z - 4)^2 = 1$.
   (a) What are the center and the radius of the sphere? (Possibly help yourself by drawing a sketch of the sphere).
   (b) Based on your answer to (a), make a guess as to what is the point on S which is closest to the origin $(0, 0, 0) \in \mathbb{R}^3$.
   (c) Write the system of equations you would need to solve to use Lagrange multipliers to find the closest point on S to the origin.
   (d) Check whether the point you guessed in (b) satisfies the system of equations you wrote in (c) (if it doesn't, go back to (b) and revise your answer).
   (e) Is there any other point on S that satisfies the system of equations you wrote in (c)? Please don't just answer yes or no: support your answer with either a

computation or a verbal argument.