HW 3
Math 261, S19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on **Friday, February 22**.

1. (5 pts.) TRUE OR FALSE
   (a) Let $h(x, y)$ be a continuous function. Then for any poiny (x_0, y_0) in the domain of h, the limit of h as (x, y) approach the point (x_0, y_0) exists.
   (b) Let $h(x, y) = x/y$. The limit of h as (x, y) approach the point $(1, 1)$ exists.
   (c) Let $h(x, y) = x/y$. The limit of h as (x, y) approach the point $(0, 0)$ exists.
   (d) Let $h(x, y) = \begin{cases} 3 & (x, y) = (0, 0) \\ 1 & (x, y) \neq (0, 0) \end{cases}$. The limit of h as (x, y) approach the point $(0, 0)$ doesn't exist.
   (e) Let $h(x, y) = \begin{cases} 3 & (x, y) = (0, 0) \\ 1 & (x, y) \neq (0, 0) \end{cases}$. The limit of h as (x, y) approach the point $(0, 0)$ is equal to 3.
2. (3 pts.) If $f(x, y, z) = \sqrt{x^3 + \sin(y) - y\ln(z)}$, find $f(2, \frac{\pi}{2}, 1)$. Perform elementary simplifications.
3. (3 pts.) Sketch the domain of $g(x, y) = \ln(1 - 2x - 2y)$.
4. (3 pts.) Let $h(x, y, z) = 3x^2z + z\cos(\pi y - \pi x) + 3e^z$. Determine $\lim_{(x,y,z)\to(1,2,0)} h(x, y, z)$.
5. (3 pts.) The function $k(x, y) = \dfrac{7x^8y}{-2x^9 + 9y^9}$ has no limit as $(x, y) \to (0, 0)$.

   Show this by computing the limit of the function along the two following paths:
   (a) $t \mapsto (t, 0)$. This notation indicates the path $(x(t), y(t)) = (t, 0)$, or equivalently, the path given by $y = 0$.
   (b) $t \mapsto (t, t)$. This notation indicates the path $(x(t), y(t)) = (t, t)$, or equivalently, the path given by $y = x$.

   Note (and hint): the nice thing about the parametric notation for the paths $t \mapsto (f(t), g(t))$ is that it suggests what you should do to compute the limit along the path: plug in the function $f(t)$ for x, the function $g(t)$ for y, and then take the limit as $t \to 0$.
6. (3 pts.) Compute $\dfrac{\partial h}{\partial x}$ for the function in #4.