

HW 9
Math 261, F19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on **Friday, November 22**.

1. (5 pts.) TRUE OR FALSE:
 - (a) If $f(x, y, z)$ is a differentiable function, then the gradient of f is a conservative vector field.
 - (b) If \mathbf{F} is a conservative vector field on a region R and C is a closed curve bounding R then the line integral of \mathbf{F} along C equals 0.
 - (c) If \mathbf{F} is a conservative vector field on \mathbb{R}^3 and C_1, C_2 two paths starting at a point P and ending at a point Q then the line integral of \mathbf{F} along C_1 equals the line integral of \mathbf{F} along C_2 .
 - (d) If \mathbf{F} is a conservative vector field, its potential function is unique.
 - (e) If \mathbf{F} is a conservative vector field, its potential function is unique up to translation by a constant.
2. (3pts.) Give an example of a conservative vector field, and of a vector field which is not conservative.
3. (3 pts.) Suppose conservative vector field \mathbf{G} has potential function $g(x, y, z) = x^2 + yz$. Compute the work done when moving through this vector field along any simple curve from from $(0, 1, 1)$ to $(2, 0, 1)$.
4. (3 pts.) Find *the* potential function $f(x, y, z)$ for vector field

$$\mathbf{F} = \langle \sin(y), x \cos(y) + z \cos(y), \sin(y) + 2z \rangle$$

such that $f(0, 0, 1) = 2$. You may assume that \mathbf{F} is conservative.

5. (3 pts.) Use the component test ($M_y = N_x$, etc.) to show that the vector field

$$\mathbf{H} = \langle ze^{xz} - \sin(x + 2y), \frac{1}{y} - 2 \sin(x + 2y) + 1, xe^{xz} + \frac{1}{z} \rangle$$

is conservative. (Your solution should consist of three equalities.)

6. (3 pts.) Consider the vector field $\mathbf{F} = \langle y, -x, 0 \rangle$. Describe (or even better, sketch) how the vector field looks like for points of the unit circle in the xy - plane. Argue, without doing the component test, that the vector field cannot be conservative.