

Finite element methods in scientific computing

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Lecture 9:

A second example:

The *step-2* tutorial program
–
Degrees of freedom (DoFs)

step-2

Step-2 shows:

- How degrees of freedom are defined with finite elements
- The *DoFHandler* class: Enrich the information stored on cells by the numbers of degrees of freedom (DoFs)
- Enumerating DoFs: *distribute_dofs()*
- Renumbering DoFs
- The sparsity pattern of a matrices
- How to visualize a sparsity pattern

step-2

Sparsity of system matrices:

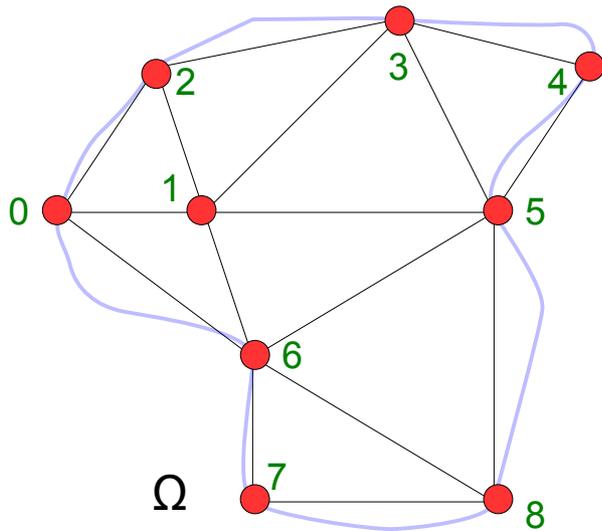
- For PDEs, finite element matrices are *always sparse*
- Result of
 - *local* definition of shape functions
 - *locality* of the differential operator

Sparsity is not a coincidence. It is a design choice of the finite element method.

Sparsity can not be overestimated as a factor in the success of the FEM!

step-2

Example: Consider this mesh and bilinear form:



$$\begin{aligned} A_{ij} &= (\nabla \varphi_i, \nabla \varphi_j) \\ &= \int_{\Omega} \nabla \varphi_i \cdot \nabla \varphi_j \, dx \end{aligned}$$

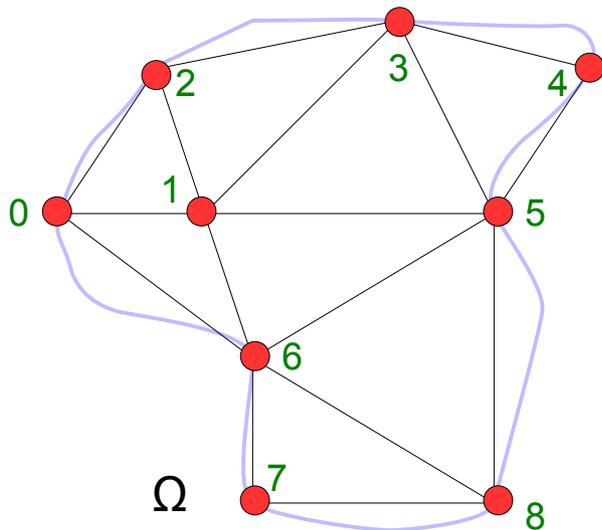
Note: In general we have that

- $A_{00} \neq 0, A_{01} \neq 0, A_{02} \neq 0, A_{06} \neq 0$
- $A_{03} = A_{04} = A_{05} = A_{07} = A_{08} = 0$

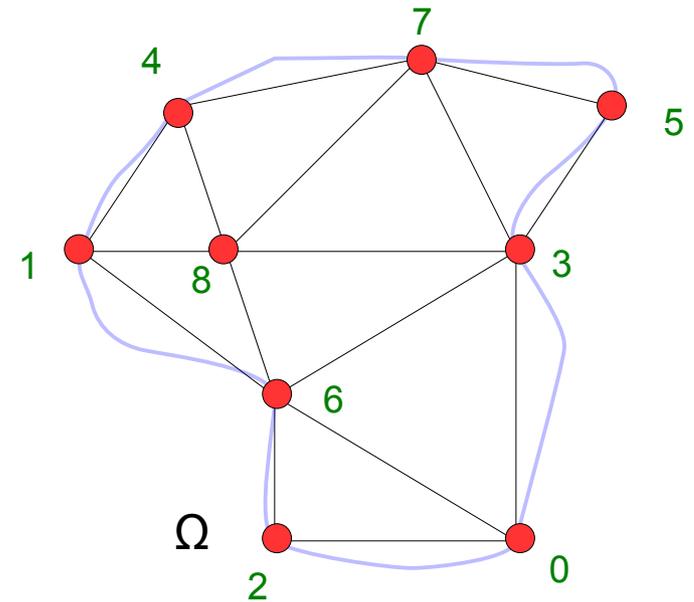
The bigger the mesh, the more zeros there are per row!

step-2

Renumbering: The order of enumerating degrees of freedom is arbitrary



VS.



Notes:

- Resulting matrices are just permutations of each other
- Both sparse; most algorithms don't care about ordering

step-2

Read through the commented program at

http://dealii.org/9.5.0/doxygen/deal.II/step_2.html

Then play with the program:

```
cd examples/step-2
```

```
cmake -DDEAL_II_DIR=/a/b/c . ; make run
```

This will run the program and generate output files:

```
ls -l
```

Then run *gnuplot* as described in the documentation

```
gnuplot
```

Next step: Play by following the suggestions in the results section. This is the best way to learn!

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