MATH 676

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Finite element methods in scientific computing

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Lecture 20:

A seventh example:

The *step-20* tutorial program (part 1)
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The mixed Laplace, a vector-valued problem

step-20

Step-20 shows:

- How to discretize a problem with more than one solution variable
- Building complicated solvers and preconditioners from simpler ones (next lecture)

step-20

Step-20 solves the mixed form of the Laplace equation:

$$\begin{aligned} K^{-1}u + \nabla p &= 0 \\ -\nabla \cdot u &= -f \end{aligned}$$

The associated weak form is

$$(\varphi_u, K^{-1}u) - (\nabla \cdot \varphi_u, p) - (\varphi_p, \nabla \cdot u) = (\varphi_p, -f)$$

step-20

From the weak form

$$(\varphi_u, K^{-1} u) - (\nabla \cdot \varphi_u, p) - (\varphi_p, \nabla \cdot u) = (\varphi_p, -f)$$

we get the matrix entries

$$A_{ij} = (\varphi_{i,u}, K^{-1} \varphi_{j,u}) - (\nabla \cdot \varphi_{i,u}, \varphi_{j,p}) - (\varphi_{i,p}, \nabla \cdot \varphi_{j,u})$$

which we transcribe into code using these substitutions:

$$\varphi_{i,u}(x_q) = \text{fe\_values[velocities].value(i,q)}$$
$$\varphi_{i,p}(x_q) = \text{fe\_values[pressure].value(i,q)}$$
$$\nabla \cdot \varphi_{i,u}(x_q) = \text{fe\_values[velocities].divergence(i,q)}$$

step-20

To implement this problem, we need to:

- Define the appropriate finite element
  – Here, use the Raviart-Thomas element for *u*, DG for *p*
  – Use *FESystem* to combine it all
- Translate the weak form into code
- Make sure output looks as expected

step-20

Read through the commented program at

http://www.dealii.org/7.3.0/doxygen/deal.II/step_20.html

Then play with the program:

cd examples/step-20

cmake -DDEAL_II_DIR=/a/b/c . ; make run

This will run the program and generate output files:

ls -l

Then visualize the solutions.

Next step: Play by following the suggestions in the results section. This is the best way to learn!

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