MATH 689: Numerical Optimization

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Homework assignment 8 – due Tuesday 3/25/2014

Problem 1 (SQP). Use the SQP method with a line search algorithm (based on an appropriate merit function) to find the solution of the following problem:

$$\min_{x\in\mathbb{R}^2} \quad \|x\| \ \arctan\|x\| - \frac{1}{2}\ln(1+\|x\|^2).$$
$$(x_1+x_2) - \sqrt{2}e^{(x_1-x_2)} - \sqrt{2}e^{-(x_1-x_2)} = 0$$

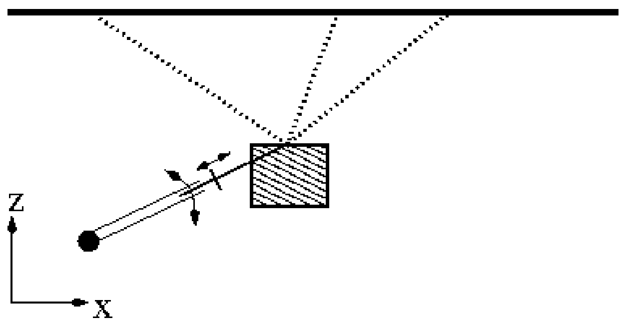
Show that you can converge from any starting point. **(4 points)**

Problem 2 (Penalty methods for inequality constrained problems). When using the quadratic penalty method for inequality constrained problems, we have to find the minimum of

$$Q_\mu(x) = f(x) + \frac{1}{\mu}\|[h(x)]^-\|^2$$

for a sequence of values $\mu \to 0$. Let's assume we want to solve the minimization of Q_μ exactly, using Newton's method with or without line search. Then we are faced with the problem that Q_μ is only in C^1, not C^3. Assuming that Newton's method converges, what convergence rate can we expect in this case (for a fixed value of μ)? Why? **(2 points)**

Problem 3 (The same old problem, once again, part 1). Consider the following system of three springs suspended from the ceiling at positions $(x, z) = (-20\text{cm}, 0\text{cm})$, $(5\text{cm}, 0\text{cm})$ and $(15\text{cm}, 0\text{cm})$ and a rod of *minimal* length 20cm that is attached at one end at $(-25\text{cm}, -20\text{cm})$ and at the other end to the same point where the springs meet each other:



Each spring has a rest length of $L_0 = 20$cm, and extending (or compressing) spring $i, i = 1 \ldots 3$ to a length L_i requires an energy of $E_{\text{spring},i} = \frac{1}{2}D(L_i - L_0)^2$ where the spring constant for all three springs equals $D = 300\frac{N}{m}$. On the other hand, the potential energy of the body is $E_{\text{pot}} = mgz$ where the body's mass is $m = 500g$, the gravity constant is $g = 9.81\frac{\text{m}}{\text{s}^2}$, and z is the vertical coordinate of the body's position.

Solve this system using (i) the quadratic penalty method, and (ii) the logarithmic barrier method for inequality-constrained problems.

(3 points)

Problem 4 (The same old problem, once again, part 2). Consider the same problem as above, but assume that the *minimal* length of the rod is now $35cm$. Solve the problem again with the two methods (penalty and barrier) and show graphically or in numbers how your solution converges. Also answer the following question, using your data (for example by computing the distance of intermediate points from the attachment point of the rod): Are intermediate iterates feasible or infeasible? **(6 points)**

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!