MATH 437: Principles of Numerical Analysis

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Bonus points homework assignment – due Tuesday 12/03/2013

Problem 1 (Numerical solution of a vector-valued ODE). Reconsider Problem 2 from last week's homework. You were asked to compute the solution to an accuracy of 100 meter, a task almost impossible with the first order forward Euler method. Solve the same problem with the fourth order Runge-Kutta method and determine the step size h you need to achieve above accuracy. **(2 bonus points)**

Problem 2 (A modeling challenge). *This problem doesn't give a whole lot of points, but if you feel bored sitting around the table with your family during Thanksgiving, do what mathematicians typically do: scribble the solution to questions like this on napkins or the back of envelopes. The problem has two parts: a theoretical part that you can do on the napkin (worth two points), and a practical part where you have to implement your model on a computer (worth only 1 point, so that if you can't do it at home it doesn't matter too much).*

Here's the theoretical problem: Thanksgiving turkeys aren't particularly good at flying. They may try, but they don't really get into the air very gracefully and for extended periods of time. Derive an ODE model for turkey flight that takes into account the following rules (all quantities have units meter, meter per second, etc, as appropriate):

a) the turkey is initially at rest;

b) it then runs horizontally, accelerating at a modest pace of 1.5;

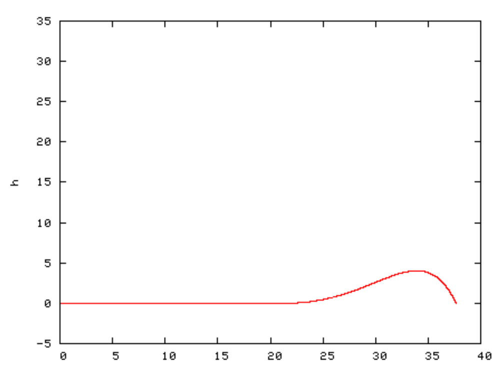
c) when it reaches the lift-off speed of $v = 8$ it gets airborne; from thereon, its vertical (upward) acceleration is $v - 4$ (in other words, the initial upward acceleration after getting airborne is 4 because its horizontal velocity was 8 at the time); at the same time, air friction reduces the horizontal velocity by a decelleration of $-v^2/10$.

The combined effect of friction and the speed dependent upward acceleration is that when the turkey's forward velocity drops below 4, vertical acceleration becomes negative and it starts to fall back to the ground.

To write an ODE model for this, you will have to use the following variables: $x(t)$–horizontal distance from the starting point; $v(t)$–horizontal velocity; $h(t)$–height above ground; $u(t)$–vertical velocity.

Practical part: Solve these equations from the turkey's start until where it falls back down to earth. Plot $x(t)$ and $h(t)$ in a single plot to show the turkey's trajectory. If you feel challenged, compute the length of the flight in both seconds and meters.

Hint: The turkey's trajectory as a $x(t)$ vs $h(t)$ plot looks like the figure below. **(3 bonus points)**



Happy Thanksgiving!