MATH 601: QUIZ 3 (9/19/2012)

NAME: UIN:

Problem 1: Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}, \qquad u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

For the following statements, determine whether they are mathematically valid and if so, compute their value:

1. $AB = \begin{pmatrix} 1 & 4 & 7 \\ 3 & 6 & 9 \\ 0 & 1 & 2 \end{pmatrix}$
2. $B^T A$: not defined because the dimensions of $B^T \in \mathbb{R}^{3\times 2}$ and $A \in \mathbb{R}^{3\times 2}$ don't match up
3. $B^T A^T = (AB)^T = \begin{pmatrix} 1 & 4 & 7 \\ 3 & 6 & 9 \\ 0 & 1 & 2 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 & 0 \\ 4 & 6 & 1 \\ 7 & 9 & 2 \end{pmatrix}$
   Note how we could re-use something that we had previously computed.
4. $\mathrm{tr}(A)$: not defined because A is not square
5. $\mathrm{tr}(AB) = \mathrm{tr} \begin{pmatrix} 1 & 4 & 7 \\ 3 & 6 & 9 \\ 0 & 1 & 2 \end{pmatrix} = 1 + 6 + 2 = 9$
6. $\mathrm{tr}(BA) = \mathrm{tr}(AB) = 9$
   Note how we again could re-use something that we had previously computed.

(see backside)

Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}, \qquad u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

For the following statements, determine whether they are mathematically valid and if so, compute their value:

7. $u \times v = \begin{pmatrix} 1 \cdot 3 - 1 \cdot 2 \\ 1 \cdot 1 - 1 \cdot 3 \\ 1 \cdot 2 - 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$
8. $u \cdot v = 1 + 2 + 3 = 6$
9. $u^T A = (4 \quad 3)$
   I.e., we interpret the result as a row vector or a 1×2 matrix.
10. Au: not defined because the dimensions of $A \in \mathbb{R}^{3\times 2}$ and $u \in \mathbb{R}^3 \simeq \mathbb{R}^{3\times 1}$ don't match up
11. $Bv = \begin{pmatrix} 14 \\ 8 \end{pmatrix}$
12. $u^T ABv$: We can rewrite this as $(u^T A)(Bv)$ where the two factors are a matrix with just one row and a matrix with just one column. The product of a row matrix and a column matrix can be interpreted as the dot product between two vectors, i.e., we can rewrite this as $(u^T A)^T \cdot (Bv) = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 8 \end{pmatrix} = 56 + 24 = 80$. Here, once more, we can reuse something we have previously computed. Of course, we could have simply multiplied everything out and arrived at the same result.
13. $u \times (Bv)$: not defined because the cross product is only defined for two vectors in $\mathbb{R}^3$ but Bv is a vector in $\mathbb{R}^2$.

Note: The quiz was graded so that questions 1, 5, 7, 8, 9, 11, 12 were counted as one point each, and questions 2, 3, 4, 6, 10, 13 with 0.5 points because the answer could re-use a result obtained earlier.