MATH 652: Optimization II

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Homework assignment 8 – due Thursday 4/8/2010

Problem 1 (An optimal control problem). If you put an amount q of fertiziler in a flower pot at time $t = 0$, watering the plant will over time reduce the available amount to $a(t) = qe^{-}t$. A plant that starts at height $h(0) = 1$ at time $t = 0$ will grow according to

$$\dot{h}(t) = a(t).$$

If we want the flower to have height $h(4) = 4$ at time $t = 4$ (for example because we want to sell it, and this height is what the customer wants), then we need to ask how much fertilizer q we need to put into the point at $t = 0$. This leads to the optimal control problem

$$\begin{aligned} \min_{h(t),q\in\mathbb{R}} \quad & \frac{1}{2}\left(h(4)-4\right)^2 = \frac{1}{2}\int_0^T \left(h(t)-4\right)^2 \delta(t-4)\,\mathrm{d}t \\ & \dot{h}(t) = qe^{-t}, \\ & h(0) = 1. \end{aligned}$$

Derive the optimality conditions for this problem and find a solution to it.

(4 points)

Problem 2 (An optimal control problem). The pilot of a plane wants to climb along a linear trajectory $\hat{z}(t) = 4t$ but there are updrafts and downdrafts that result in vertical forces $\cos(t)$. We will need to produce forces $q(t)$ so that the plan climbs along the desired route. Let's model this problem as follows:

$$\begin{aligned} \min_{z(t),q(t))} \quad & \int_0^T \left(z(t)-\hat{z}(t)\right)^2 \mathrm{d}t \\ & \ddot{z}(t) = \cos(t) + q(t), \\ & z(0) = 0, \\ & \dot{z}(0) = 4. \end{aligned}$$

Reformulate the problem so that you have only first-order time derivatives by introducing a second variable $v(t)$ as in class. Formulate optimality conditions and find a solution to the optimization problem based on these. **(6 points)**

Problem 3 (More on this). Describe what would have happened if the second initial condition had been

$$\dot{z}(0) = 0.$$

What would the optimality conditions have been in that case, and what does that mean for the solution. **(2 points)**

Problem 4 (Even more on this). Can you come up with a way how we could have worked with the originally posed formulation of Problem 2, i.e. without introducing a velocity variable $v(t)$? Hint: You need a variational form of the differential equation that contains two time derivatives, along with their initial conditions. **(2 points)**

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!