MATH 442: Mathematical Modeling

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Homework assignment 2 – due 9/16/2010

Problem 1 (Estimating parameters in models). When we model processes, we often say things like this: Let's say the growth rate is a and the starting point at time t_0 is b. Then our model predicts that at time t the number of bacteria in the petri dish is $y(t) = be^{at}$. This is all well and good, and we can do all sorts of nice mathematical analyses with this. But at one point we want to actually predict the number of bugs in the petri dish for $t =$tomorrow afternoon. To do this, we need to know a, b. We typically obtain these values by considering values y_i that were actually measured at times t_i in the past and *fitting* parameters a, b so that the resulting prediction $y(t_i)$ best fits the measured values y_i. Let's do this for a simple case.

Go to the US Census Bureau website and obtain the total population in the United States for each of the years 1900, 1910, 1920, ..., 2000. Show these values graphically and in numbers. **(2 points)**

Next, determine the best parameters for each of the following models by fitting them to the data above using a Least Squares procedure:

- Assumed linear population growth: $y(t) = at + b$;
- Assumed exponential population growth: $y(t) = ae^{bt}$;
- Assumed logistic population growth: $y(t) = \frac{ab}{a+(b-a)e^{-rt}}$.

For each case, plot the function $y(t)$ (where you take the best fits for the parameters obtained from the least squares fit) along with the actual population numbers you looked up before. Document how you arrive at your answers by attaching your Maple worksheet. **(8 points)**

For each of the three models above and the respectively best set of parameters, determine the quality of the model by evaluating the misfit. For example, if the y_i are the population numbers at times t_i, then for the first of the models above,

$$\text{misfit} = \sqrt{\frac{1}{11} \sum_{\substack{i=1900 \\ \text{in increments of 10}}}^{2000} [y_i - y(t_i)]^2}$$

$$= \sqrt{\frac{1}{11} \sum_{\substack{i=1900 \\ \text{in increments of 10}}}^{2000} [y_i - (at_i - b)]^2}$$

Compute these values for the three models and interpret your results. Indicate which of the models provides the best and which the worst fit. **(6 points)**

Problem 2 (Solving ODEs with Maple). Consider the initial value problem

$$y'(t) = r\frac{K - y(t)}{K} y(t),$$
$$y(0) = y_0.$$

(This differential equation is called the "logistic equation"). Using Maple, find its analytic solution. Plot the solution on the interval $t = 0...2$ for the particular values $K = 10, r = 1, y_0 = 1$. What are the values of $y(t)$ at times $t = 1$ and $t = 20$ to four digits of accuracy?

Show your work as a Maple document or worksheet. **(6 points)**

Bonus problem. Repeat problem 1 with the population numbers of Texas instead of the United States. **(4 bonus points)**

I try to be as good a teacher as possible, but to succeed in this goal I need feedback from those who see me teach, i.e. you. If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, group work vs. whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!