MATH 442: Mathematical Modeling

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Homework assignment 1 – due 9/9/2010

Problem 1 (What fascinates and excites you?). At any given time, I always have a few questions going around in my mind that I would really like to understand. These are things that I find fascinating and interesting but for which I lack the knowledge to give definite answers. Here are just two out of many: (i) Why are continents moving around, instead of staying put? Is it the friction of material in the earth mantle moving around under them? Is it the drag of cold dense parts of these plates that have subducted into the hotter, less dense earth mantle and are sinking? Or maybe the pressure of material pushing up at mid-ocean ridges? What are the factor I need to understand to estimate the relative strengths of these effects? (ii) For most organisms, regular cells have two copies of each chromosome, but when they divide to form gametes (meiosis), only one copy of each chromosome ends up in each sperm or egg cell. Since the individual inherited the two chromosomes from its mother and father, respectively, you'd think that the sperm or egg cell should contain a chromosome that is equal to either the chromosome received from the mother or father, but this isn't so: it's actually a mixture and the process that produces it is called crossover, a form of genetic recombination. The question is: How does this happen on the level of the DNA, what prevents crossover to happen in the middle of a gene, and how do all these factors affect the random mixing of genes received from the parents and passed on to the children?

You must have such questions as well: Something that you find fascinating but that seems a bit out of your league with your current knowledge. Describe two such questions from physics, chemistry, biology, economics, ..., in half a page or more each and discuss what role mathematics could play in providing answers. Also identify for each of the two problems what skills or what kind of knowledge you are currently lacking to address them. **(8 points)**

Problem 2 (What are you good at?). Mathematical modeling is an area that requires a broad set of skills:

1. The ability to concisely describe in words a mental concept. For example: "Friction is the force two bodies exert on each other as they move past each other while in contact."

2. The ability to find the critical parts of such statements and identify their properties. For example: "The central concept of friction is *force*, which

is a vector with direction and magnitude."

3. Insight into the problem at hand (whether it be physical, biological, behavioral, etc.) and intuition. For example "If the two bodies moving past each other have smooth surfaces, the friction force direction should be parallel or antiparallel to their relative velocity. The faster they move past each other the larger we expect the force's magnitude to be. The force's magnitude will also be larger the harder the two bodies are pushed together and the rougher the surfaces are."

4. The ability to translate these verbal statements into formulas. For example: "A first approximation of the principles above would imply a linear relationship of force and velocity: $\mathbf{F}_{\text{friction}} = \alpha \mathbf{v}_{\text{relative}}$, where α is a proportionality constant that depends on the various factors listed above."

5. The ability to analyze the resulting relationships for their properties, such as whether they make physical sense. For example: "The equation above allows for the conservation of linear momentum, whereas an equation that contained the *absolute* rather than *relative* velocity would not conserve linear momentum."

6. The knowledge on where to find or otherwise acquire values for the various parameters in equations, such as the constant α above.

7. The ability to solve the equations, given known inputs (such as $\alpha, \mathbf{v}_{\text{relative}}$), for the desired outputs (such as $\mathbf{F}_{\text{friction}}$) in cases where the equations are a bit more complicated.

Oftentimes, engineers are really good at points 1–3 and 6, physicists at point 4, and mathematicians at points 5 and 7.

Rate on a scale from 1 (very good) to 5 (not so much) how you estimate your skills in each of the points above. Describe in a paragraph each the point that you expect to be the most fun to you and the point with which you expect the most trouble this semester, and in each case explain why (for instance giving an example from previous semesters). **(4 points)**

I try to be as good a teacher as possible, but to succeed in this goal I need feedback from those who see me teach, i.e. you. If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, group work vs. whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!