MATH 651: Optimization I

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Homework assignment 4 – due Thursday 10/8/2009

Problem 1 (Trust region algorithm using the dogleg strategy). In class we discussed the dogleg algorithm to determine the step $\tilde{p}_k$ in a trust region algorithm. The final slide discussed how to choose the step if Δ_k is small, medium, or large, in which case we either choose the "unconstrained minimizer" the in steepest descent direction, a point between the unconstrained minimizer and the quasi-Newton update, or the quasi-Newton update, respectively. However, this strategy wouldn't work if the dogleg would intersect the trust region sphere in more than one point.

Prove that that can't happen, i.e. that the dogleg curve has only a single intersection point with the trust region sphere. For the proof you will have to assume that B_k is chosen as a positive definite matrix. **(4 points)**

Problem 2 (A nontrivial example). Implement a line search Newton algorithm to find the minimum of the following function $f : \mathbb{R}^N \to 0$,

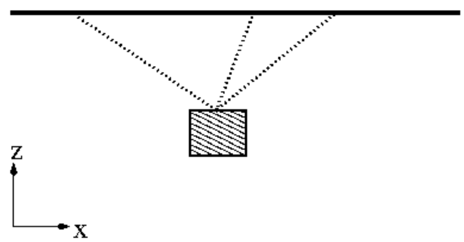
$$f(x) = \sum_{i=1}^{N} \left[(1 - x_{2i-1})^2 + 10(x_{2i} - x_{2i-1}^2)^2 \right],$$

with $N = 10$. Show convergence by providing a representative sample of your iterates x_k as well as your best guess of the location of the minimum x^*, along with the function values of $f(\cdot)$ at all of these points. Verify with the necessary conditions that you have indeed found a minimum.

This is a challenging problem, and you may have have to choose your starting point intelligently to get convergence. Unless you are absolutely certain that you've found the minimum, provide your source code to get at least partial credit. **(8 points)**

Can you use your program to also find the minimum of $f(x)$ if $N = 50$? $N = 100$? $N = 1000$? **(up to 2 bonus points)**

Problem 3 (A modeling exercise). Consider the following system of three springs suspended from the ceiling at positions $(x, z) = (-20\text{cm}, 0\text{cm})$, $(5\text{cm}, 0\text{cm})$ and $(15\text{cm}, 0\text{cm})$ and that hold in place a body:



Each spring has a rest length of $L_0 = 20\text{cm}$, and extending (or compressing) spring $i, i = 1 \ldots 3$ to a length L_i requires an energy of $E_{\text{spring},i} = \frac{1}{2}D(L_i - L_0)^2$ where the spring constant for all three springs equals $D = 300\frac{N}{m}$. On the other hand, the potential energy of the body is $E_{\text{pot}} = mgz$ where the body's mass is $m = 500g$, the gravity constant is $g = 9.81\frac{\text{m}}{\text{s}^2}$, and z is the vertical coordinate of the body's position.

Express the total energy in the system (spring energies plus potential energy) as a function of the body's position (x, z). Make sure to convert all the quantities above into common units kg, m, and s. Nature likes to do things so that the energy is minimal, so use a numerical method to find the location at which this energy is minimal. In your answer, show this location as well as the value of the total energy function at this location. **(4 points)**

Problem 4 (To converge or not to converge). Consider the function $f(x) = x_1^4 - x_1^2 + x_2^2$. It's minima lie at $x^* = (\pm\frac{1}{2}\sqrt{2}, 0)^T$. Explain in words, graphs, or numbers what is going to happen if we started Newton's method at the point $x_0 = (0, 2)^T$ and did all computations exactly (i.e. not in floating point arithmetic), and explain why this leads to a somewhat unsatisfactory result. **(3 points)**

If you have comments on the way I teach – in particular suggestions how I can do things better, if I should do more or less examples, powerpoint slides vs whiteboard, etc – or on other things you would like to critique, feel free to hand those in with your homework as well. I want to make this as good a class as possible, and all comments are certainly much appreciated!