MATH 417: Numerical Analysis

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Homework assignment 6 – due 3/22/2007

Problem 1 (Jacobi iteration). Let A, b be the 100×100 matrix and 100-dimensional vector defined by

$$A_{ij} = \begin{cases} 2.01 & \text{if } i = j, \\ -1 & \text{if } i = j \pm 1, \\ 0 & \text{otherwise,} \end{cases} \qquad b_i = \frac{1}{100} \sin\left(\frac{2\pi i}{50}\right).$$

Apply Jacobi's method to solving $Ax = b$. Write a program that implements the Jacobi method and start with a vector x_0 with randomly chosen elements in the range $-1 \le (x_0)_i \le 1$ (i.e. with elements generated from what the `rand()` function or a similar replacement returns).

(Hint: It is not necessary to actually store the complete matrix just to multiply with it. Rather, use that the i-th component of the vector Ay is $(Ay)_i = \sum_{j=1}^n A_{ij} y_j = 2.01 y_i - y_{i-1} - y_{i+1}$ at least for $2 \le i \le n-1$, and obvious modifications for $j = 1$ and $j = n$.)

Run 200 Jacobi iterations and plot the values of $(x^{(k)})_i$ against i for every few iterations, for example $k = 0, 2, 5, 10, 20, 50, 100, 200$. What do you observe? **(5 points)**

Problem 2 (Alternative vector norms). Let A be a symmetric and positive definite $n \times n$ matrix. Show that

$$\|x\|_A = \sqrt{x^T A x}$$

is a norm for vectors $x \in \mathbb{R}^n$. (Hint: Use the eigenvalue and eigenvector decomposition of symmetric positive definite matrices.)

(3 points)

Problem 3 (Jacobi iteration). Solve problems 7.3.1 a) and b) of the book (using paper and pencil). **(3 points)**

Problem 4 (Gauss-Seidel iteration). Solve problems 7.3.3 of the book (using paper and pencil) for parts a) and b). **(3 points)**

Happy spring break!