# MATH 412: Theory of Partial Differential Equations

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## Homework assignment 3 – due Thursday 9/20/2007

**Problem 1 (Linear operators).** Solve exercise 2.2.2 in the book.
**(2 points)**

**Problem 2 (Linear operators).** Solve exercise 2.2.5 in the book.
**(3 points)**

**Problem 3 (Sign of eigenvalues).** Consider the equation

$$\frac{d^2\phi(x)}{dx^2} + \lambda\phi(x) = 0.$$

Determine whether $\lambda > 0, \lambda \geq 0, \lambda \leq 0$, or $\lambda < 0$ based on the technique shown in class, for each of the following sets of boundary conditions:

$$\begin{aligned} &a) \qquad \phi(0) = \phi(\pi) = 0, \\ &b) \qquad \frac{\partial\phi}{\partial x}(0) = \frac{\partial\phi}{\partial x}(1) = 0. \end{aligned}$$

**(3 points)**

**Problem 4 (Orthogonality of trigonometric functions).** Solve exercise 2.3.5 in the book. **(2 points)**

**Problem 5 (Eigenfunctions of $\frac{\partial^2}{\partial x^2}$).** As part of solving the heat equation for one space dimension, we had to find the solutions of the equations

$$\frac{\partial^2\phi(x)}{\partial x^2} = -\lambda\phi(x), \qquad \phi(0) = 0, \qquad \phi(L) = 0.$$

The (non-trivial) solutions were $\phi_n(x) = \sin(n\pi x/L)$, for $n = 1, 2, \ldots$ Repeat this exercise by finding the solutions of the eigenproblem

$$\frac{\partial^2\phi(x)}{\partial x^2} = -\lambda\phi(x), \qquad \phi(0) = 0, \qquad \frac{\partial\phi}{\partial x}(L) = 0,$$

where only the boundary condition at the right has been changed. **(4 points)**

**Problem 6 (Solutions of the heat equation).** Solve problem 2.3.3 (all parts) in the book. Note the remark at the top of the next page and that similar problems are solved in the main text. **(4 points)**