MATH 417: Numerical Analysis

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Homework assignment 9 – due 11/9/06 and 11/13/06

Problem 1 (Lagrange interpolation). For the data set $x_i = \{1, 2, 3, 4, 5\}$, $y_i = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$, compute the Lagrange interpolation polynomial. Plot this polynomial together with the function $f(x) = \frac{1}{x}$ and describe where the interpolating polynomial is a reasonable approximation of $f(x)$.

(3 points)

Problem 2 (Lagrange interpolation of higher order). For each of the values $N = 1, 2, 4, 6, 8, 12, 20$, compute the polynomial $p_{2N}(x)$ of order $2N$ such that

- $p_{2N}(0) = 1$,
- $p_{2N}(\pm\frac{j}{N}) = 0$ for $j = 1, \dots, N$.

Plot these polynomials in the interval $-1 \le x \le 1$. What happens as N becomes larger? (Hint: You will want to compute the polynomials with a computer algebra system or a self-written program, since computing polynomials of degree 40 on paper becomes tedious. You can make your life a lot easier by only computing those polynomials that you actually need.) **(6 points)**

Problem 3 (Non-equidistant Lagrange interpolation). Modify your program for Problem 2 to solve the interpolation problem

- $p_{2N}(0) = 1$,
- $p_{2N}\left(\sin\left(\pm\frac{\pi j}{2N}\right)\right) = 0$ for $j = 1, \dots, N$

for all values of N in problem 2. Note that the interpolation points $\sin\left(\pm\frac{\pi j}{2N}\right)$ are between -1 and 1 as before, but are now no longer equidistantly spaced.

(3 points)

Problem 4 (Numerical differentiation). In class, the symmetric second difference quotient

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

was introduced. Here, we want to study its properties.

(a) Compute the quadratic Lagrange interpolation polynomial $p_2(x)$ that interpolates f in the points $x-h$, x and $x+h$ and show that the formula is the second derivative $L''(x)$ of this polynomial.

(b) Show that the formula is exact for all polynomials of degree at most 3 (Hint: show this for the monomials $x^k, k = 0, 1, 2, 3$ and explain why this is sufficient).

(c) Use the Taylor polynomial of degree 3 for f around the point x and its remainder term to show that

$$f''(x) - \frac{f(x-h) - 2f(x) + f(x+h)}{h^2} = -\frac{h^2}{12} f(4)(\xi)$$

for some $\xi \in (x-h, x+h)$. **(6 points)**