MATH 417: Numerical Analysis

Instructors:	Prof. Wolfgang Bangerth, Prof. Guido Kanschat
	`bangerth@math.tamu.edu`,
	`kanschat@math.tamu.edu`
Teaching Assistants:	Seungil Kim, Yan Li
	`sgkim@math.tamu.edu`,
	`yli@math.tamu.edu`

Homework assignment 2 – due 9/14/06 and 9/15/06

Problem 1 (Fixed-point iteration). Let a be a positive number and

$$g(x) = 2x - ax^2.$$

(a) Show that the limit of this fixed-point iteration is either 0 or $1/a$.

(b) This iteration gives us a means to compute the reciprocal value of a using only multiplication and subtraction. Find an interval about $1/a$ in which this iteration converges to $1/a$. **(6 points)**

Problem 2 (Bisection method). A ball is dropped from a tower. During its fall it is subject to the forces of gravity and air resistance. Thus, the height h in ft as a function of time t in s is given by the formula

$$h(t) = h_0 - \frac{mg}{k}t + \frac{m^2g}{k^2}\left(1 - e^{-\frac{kt}{m}}\right).$$

Here, $h_0 = 300 ft$ is the height of the tower, $m = 1/2 lb$ the mass of the ball, $g = 32.17 ft/s^2$ is the gravitation of earth and $k = 1/4 \, lb \, s/ft$ is the air resistance coefficient.

Use the bisection method (on paper, with pen and calculator) to find out when the ball hits the ground with an accuracy of 0.1 seconds. **(6 points)**

Problem 3 (Regula falsi). This scheme can be obtained from the bisection method by not using the midpoint of the interval $[a_n, b_n]$ to subdivide it into two parts, but instead using the point

$$\frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$$

and continuing the method as before, i.e. choosing either the left or the right part of the interval. Show that this point is where the the secant of the function between the points a and b crosses the x-axis. Show that it is indeed within the interval (a_n, b_n).

Program this method and compare it to the bisection method you worked on in the lab to compute the roots of

(a) $x^2 - 2$ in $[0, 2]$

(b) $\cos x$ in $[0, 3]$

(c) $x^{10} - 0.1$ in $[0, 1]$

to an accuracy of 10^{-6}. Plot or draw the estimated errors in each step of both methods in a diagram. **(8 points)**