MATH 412: Theory of Partial Differential Equations

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Homework assignment 3 – due Thursday 9/21/2006

Problem 1 (Bivariate analysis). Here is a picture of the large radio telescope in Arecibo, Puerto Rico:

Impose a coordinate system with the origin at the center of the dish and such that the positive x-axis runs from the origin in the direction of the tower in front. Let Ω be the domain in x-y-space occupied by the dish. Let $H(x, y)$ be the height of the telescope's surface above the level defined by the circular rim (the surface is of course below the rim, so $H(x, y) \leq 0$).

a) Plot the coordinate system (i.e. x- and y-axes) into the picture. Indicate $H(0, 0)$.

b) Describe in words the meaning of the following quantities defined on the entire domain and state the sign of the quantities on the second line:

$$\frac{\partial H(x,y)}{\partial x} \qquad \frac{\partial H(x,y)}{\partial y} \qquad \nabla H(x,y)$$

$$\frac{\partial^2 H(x,y)}{\partial x^2} \qquad \frac{\partial^2 H(x,y)}{\partial y^2} \qquad \Delta H(x,y)$$

$$\int_\Omega H(x,y)\,dx\,dy \qquad \int_{-R}^{R} H(x,0)\,dx \qquad \nabla H(0,0)$$

c) Describe in words the meaning of the following quantities defined on the boundary of the domain and state the sign of quantities where possible:

$$\mathbf{n} \qquad \frac{\partial H(x,y)}{\partial n} \qquad \frac{\partial^2 H(x,y)}{\partial n^2}$$

$$\int_{\partial\Omega} H(x,y)\,ds \qquad \int_{\partial\Omega} \frac{\partial H(x,y)}{\partial n}\,ds$$

(5 points)

Problem 2 (Eigenfunctions of $\frac{\partial^2}{\partial x^2}$). As part of solving the heat equation for one space dimension, we had to find the solutions of the equations

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -\lambda \phi(x), \qquad \phi(0) = 0, \qquad \phi(L) = 0.$$

The (non-trivial) solutions were $\phi_n(x) = \sin(n\pi x/L)$, for $n = 1, 2, \ldots$ Repeat this exercise by finding the solutions of the eigenproblem

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -\lambda \phi(x), \qquad \phi(0) = 0, \qquad \frac{\partial \phi}{\partial x}(L) = 0,$$

where only the boundary condition at the right has been changed. **(4 points)**

Problem 3 (Solutions of the heat equation). Solve problem 2.3.3 (all parts) in the book. Note the remark at the top of the next page. **(4 points)**