frivolous (wrong accent in *Adéle* and *Adéle group*) to the serious (the definition of *Galoisian* omits separable). But, when I polled my colleagues, the geometers and topologists among them alerted me to something much worse. *Zariski topology* is defined as if the affine line (over an infinite field) were the only space there is! The *Riemann tensor*, strictly speaking, is not a tensor; much of the treatment of Riemannian geometry relies on books addressed to engineers, where formulas dominate concepts. *Riemannian metric* only considers compact manifolds, *topological spaces* might mislead you into thinking they must all be Hausdorff . . . , the list could go on. Sadly, in this respect the current web version is only worse: the entry for Riemann–Roch (not yet in print) is muddled and the classification of *compact manifolds* (ditto) is plain wrong.

Unusual. The encyclopedia corrects errors, e.g., under Zipf's law (p. 1968), Pierce's statement that $\sum P(r) > 1$ for $r = 8727$ is incorrect.

Weight. At 4.16 kilos (I weighed it in the Bonn post office; it would cost you 6.70 Euros to mail it), it weighs almost as much as I did when I was born (I happen to know this because it is the only reason for pride I have ever given my mother in my entire life) but I carried it jubilantly across five different countries this summer. Somehow holding this slick volume full of well-lived-with gems is an antidote to loneliness, cynicism, political bleakness, and the weather. Just please don't drop it, as I did a calculus book once, on your Dean's foot—if you are still untenured, that is. But, to go back to our beginning: if *Stat rosa pristina nomine. Nomina nuda tenemus* [E], then this book, at little less than 11,432 roses (for that's actually the current `mathworld` count), is a bargain indeed.

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Adaptive Finite Element Methods for Differential Equations. *By W. Bangerth and R. Rannacher.* Birkhäuser-Verlag, Basel, 2003. $29.95. viii+207 pp., softcover. ISBN 3-7643-7009-2.

Over the past half-century, the finite element method has emerged as the method of choice for the numerical approximation of elliptic boundary value problems, particularly those arising in structural mechanics and particularly among the engineering community. The finite element method is now routinely applied as a tool for the numerical approximation of solutions of differential equations in fields ranging from engineering and physical sciences to computational finance.

Traditionally, the finite element method was applied on either a single, or sometimes a sequence of, roughly uniform meshes in conjunction with polynomials of low order leading to a direct approximation of the primal variables throughout the computational domain. Frequently, the true solutions of the differential equations exhibit strongly localized features whose resolution might easily require significantly more degrees of freedom than are needed in regions where the solution is more well behaved. It is therefore natural to attempt to tailor the mesh to reflect the nature of the particular solution of the particular problem under consideration, and many meshes used in early studies of the 1960s and 1970s incorporate some degree of grading based on the practitioner's expectation and experience of where mesh refinement is likely to be beneficial. During the late 1970s and early 1980s, in recognition of the fact that such a priori mesh refinements are generally overly conservative, adaptive feedback procedures were developed based on computable a posteriori estimates of the error, usually measured in an energy-type norm. Here, starting from an initial and often rather coarse mesh, a sequence of meshes are obtained adaptively through systematically refining or subdividing those elements whose contributions to the global error estimate is largest while coarsening or derefining elements whose contribution is small.

In many applications, this type of approach was found to produce optimal rates of decay of the error despite the presence of singularities, interfaces, and boundary layers. Despite considerable computational experience, it was only comparatively recently proved, and then only in the case of approximation of the Poisson equation in two dimensions, that such feedback procedures really do deliver optimal rates of convergence in general.

A characteristic of such procedures has been that feedback was controlled using an a posteriori estimate of the error throughout the computational domain. Nevertheless, as is often the case in mathematics, the study of functionals of the solution is of more interest than the primal variable itself. Many examples naturally spring to mind, such as heat flux in thermal simulations, lift and drag coefficients in fluid simulations, and stress intensity factors in linear elastic fracture, to name but a few. Indeed, in many cases the sole goal of the simulation is to approximate the value of a single number corresponding to a particular functional of the solution.

It is therefore quite natural to attempt to target adaptive refinements towards the goal of the approximation of the main quantity of interest. This is the subject matter of the book by Bangerth and Rannacher. Of course, this idea is not new and there is a long history in the finite element literature of approaches geared towards deriving computable estimates and bounds for functionals of the primal unknown, going at least as far back as the work of Babuška and Miller in the early 1980s.

Roughly speaking, the main idea is based on the fact that the error in the quantity of interest may be expressed as a sum of contributions from individual elements. The individual contributions take the form of residuals (obtained by inserting the discrete, finite element approximation into the weak form of the equations) multiplied by weights involving an *influence function*, given by the solution of the dual variational statement with the quantity of interest as data. Early approaches typically sought to derive guaranteed upper bounds on the error. Such stringent requirements not only limit the applicability of the approach, but often lead to overly pessimistic bounds on the error. In recent years, the group headed by Rannacher has advocated a more liberal approach whereby the requirements for guaranteed bounds are relaxed and the emphasis shifted towards obtaining realistic estimates of the error using an approach based on *dual weighted residuals* (DWR), whereby estimates for the weights are obtained by a finite element approximation of the influence function. A key observation is that relatively crude approximations of the weights may still give realistic estimates for the overall error in the quantity of interest.

The current book is comprised of the notes from a series of lectures on this topic presented by the second author at ETH Zürich in 2002. The emphasis is very much on indicating the flexibility of the DWR approach through discussion of a range of applications and accompanying numerical examples. The basic approach is first illustrated in the context of simple linear elliptic PDEs and even ODEs where, perhaps surprisingly, the approach even offers new perspectives on the classical subject of error control for initial value problems. The ideas are then revisited in the more abstract setting of nonlinear variational problems. The remaining chapters are largely independent and illustrate the approach for PDE eigenvalue problems, optimal control and parameter estimation, parabolic and hyperbolic PDEs, and applications in structural and fluid mechanics.

The later chapters draw heavily on individual Ph.D. and Diploma theses produced in Rannacher's group, which gives some indication of the level of the presentation. In fact, very little in the way of prerequisites beyond basic knowledge of finite element approximation and familiarity with the application area is assumed. Most graduate students in engineering and physical sciences should be able to handle the material without excessive difficulty. The presentation is very much a tutorial approach promoting a hands-on experience, reinforced with practical exercises at the end of each chapter, aimed towards practitioners. The reader will not find much in the way of theoretical support in the book for the methodology proposed here, although there is a short chapter devoted to this, mainly because of the scarcity of mathematical justification in the literature. Much of the material presented in the book can be found in various survey articles by the second author, of which [1] is perhaps the most accessible. While the more seasoned practitioner will probably prefer the presentation in [1, 2], the present book provides a gentler introduction for the beginning graduate student or nonspecialist practitioner.

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Mathematics in Population Biology. *By Horst R. Thieme.* Princeton University Press, Princeton, NJ, 2003. $49.50. xx+543 pp., softcover. ISBN 0-691-09291-5.

As the title suggests, the book *Mathematics in Population Biology* stresses the mathematics behind the models of population biology. For each model, the mathematical assumptions are clearly stated and the solution behavior rigorously verified in a theorem/proof format. The models studied in this book are deterministic, formulated primarily as either ordinary differential equations or as first-order partial differential equations. However, there is one chapter devoted to scalar difference equations. The chapters are not organized according to mathematical topic but according to biological model or biological principle. The first models discussed are for simple, single-species populations, but the models progress to more complex structured models. The book is divided into four parts. Part 1 covers single-species population growth models, Part 2 covers stage-structured models with demographics, and Part 3 covers infectious disease models. Part 4 is a toolbox of mathematical techniques and tools useful to Parts 1–3. Part 4 is divided into three appendices: Appendix A, Ordinary Differential Equations; Appendix B, Integration, Inte-