## Exam Policy

(i) **No** calculator, textbook, homework, notes, or any other references should be used. Please write down all necessary steps, partial credit will be given if deserved.

(ii) You could use two double-sided Cheat Sheets for this exam.

*Good luck!*
Problem 1. True or False, circle your answer (2 points for each item, no partial credit).

(i) (T) (F) The function \( y(t) = e^{-3t} \sin(2t) \) is a solution of the 3rd order ODE \( y'''(t) + 7y''(t) + 19y'(t) + 13y(t) = 0 \).

(ii) (T) (F) The ODE \( x \sin(y)dx + (1 - x^2 \cos(y))dy = 0 \) is exact.

(iii) (T) (F) If \( v \) is an eigenvector of matrix \( A \) associated with the eigenvalue \( \lambda_0 \), then \( (A - \lambda_0 I)^2 v = 0 \).

(iv) (T) (F) \( x(t) = e^{2t} \) is a particular solution of \( x'''(t) - 3x'(t) - 4x(t) = 3e^{2t} \).

(v) (T) (F) For the given matrix \( A = \begin{bmatrix} e & 0 & 0 \\ 0 & \pi & 1 \\ 0 & 0 & \pi \end{bmatrix} \), the algebraic multiplicity of the eigenvalue \( \lambda = \pi \) is two.

(vi) (T) (F) For the given matrix \( A = \begin{bmatrix} e & 0 & 0 \\ 0 & \pi & 1 \\ 0 & 0 & \pi \end{bmatrix} \), the geometric multiplicity of the eigenvalue \( \lambda = \pi \) is two.

(vii) (T) (F) Consider the ODE system \( \begin{cases} x'(t) = x - y \\ y'(t) = x + y \end{cases} \). The equilibrium point at the origin is a center.
Problem 2. Find a particular solution of $x''(t) - 3x'(t) - 4x(t) = e^{-t}$. 

(10 points)
Problem 3. Consider the 1st order ODE \( \cos(t)x'(t) - \sin(t)x(t) = \sec^2(t) \) and an initial condition \( x\left(\frac{\pi}{4}\right) = -\sqrt{2} \).

(i) Find the general solution to the ODE.

(ii) Find the solution satisfying the initial condition.
Problem 4. Consider an ODE system \[
\begin{align*}
x'(t) &= y + x(2015^2 - x^2 - y^2), \\
y'(t) &= -x + y(2015^2 - x^2 - y^2).
\end{align*}
\]

(i) Show that \(x(t) = 2015 \sin(t), y(t) = 2015 \cos(t)\) is a solution of the ODE system.

(ii) Is there any solution curve passing through the point \((0, -1)\)? How many? Justify.

(iii) Assume that \(\Gamma\) is a solution curve in part (ii), prove that the distance from the origin to any point on the curve \(\Gamma\) is less than 2015.
(16 points) Problem 5. Given an ODE system $\mathbf{x}'(t) = A\mathbf{x}$, where $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, find the real-valued general solution.
Problem 6. Given an ODE $x''(t) - 3x'(t) = 4\sin(t)$,

(i) Use the method of undetermined coefficients to find one particular solution.

(ii) Find the general solution.
Problem 7. Apply Laplace and inverse Laplace transforms to solve the IVP:
\[ y''(t) - 5y' + 6y = e^{3t}, \quad y(0) = 0, \quad y'(0) = 1. \]