Homework 2
Renzo's math 472

1 Life with a metric

Exercise 1. Which of the following is a metric function on $\mathbb{R}^n$? Explain.
For $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n)$

$$d(x, y) =$$

1. $\max_{i=1,\ldots,n} |x_i - y_i|$ 
2. $|x_1 - y_1|

Exercise 2. What are limit points for sets in the discrete metric?

Exercise 3. Prove that a set is open/closed in the euclidean metric iff it is open/closed in the taxi-cab metric.

Exercise 4. What are open and closed sets in the discrete metric?

Exercise 5. Assume $X$ is a metric space. Choose ONE of the following statements and prove it.

- $\phi, X$ are open/closed.
- the arbitrary union of open sets is open.
- the finite intersection of open sets is open.
- complement of an open is closed and viceversa.

2 Topology: no more metric.

Exercise 6. Describe all possible topologies on a set $X$ consisting of three points.

Exercise 7. Which of these are topologies on $\mathbb{R}$?

- $A = \{\phi, \mathbb{R}, (-\infty, a) \text{ for all } a \in \mathbb{R}\}$
- $B = \{\phi, \mathbb{R}, (a, +\infty) \text{ for all } a \in \mathbb{R}\}$
Exercise 8. Let $X$ be a topological space with the stupid topology, and $x \in X$ a point of $X$. For what subsets of $X$ is $x$:

- an interior point?
- a limit point?

What is the smallest neighborhood of $x$?