Beware the mathematician and others who make false prophecies. The danger already exists that the mathematicians have made a pact with the Devil to darken the spirit and confine Humans within the bonds of Hell.  -St. Augustine

1. Read F9, F10, F11.


3. * Prove that $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \mod mn$ if $m$ and $n$ are relatively prime.

4. The number of keys for the Caeser shift cipher on an alphabet of $n$ letters is $n - 1$. In other words, the number of non-trivial ways of encoding by shifting by between 1 and $n$ letters is $n - 1$. Find the number of non-trivial keys for the following types of ciphers:

   i) a monoalphabetic substitution cipher on an alphabet of $n$ letters;
   ii) an affine cipher on an alphabet of $n$ letters;
   iii) a Vigenere cipher using words of length $n$.

5. Look at Bhuvana’s table of periods of the Fibonacci sequence mod $m$. Make two careful conjectures about the periods $\pi(m)$. Try to prove one (hint-maybe using the Sun Ze (Chinese Remainder) Theorem).