Mathematics V1205x, Calculus IIIS/IVA
Sample Midterm 2 Answers: Spring 2003

1. Suppose $F = 2xyi + (x^2 + nyz)j + y^2k$.
   A. For what number $n$ does $\text{curl}(F) = 0$?
      $n = 2$
   B. For this number $n$, find $f$ so that $\nabla f = F$.
      $f = x^2y + y^2z$

2. Let $F = (1 + \tan(x), x^2 + e^y)$ be a force field. Let $C$ be the boundary of the region enclosed by the parabola $x = y^2$ and the lines $x = 1$ and $y = 0$. Find the work done by $F$ as a particle travels once around $C$ in the counterclockwise direction.
   $W = 4/5$

3. Find a vector field $F$ such that $\int_C F \cdot dr = 0$ whenever the endpoints of $C$ both lie on the curve $y = x^3 + x + 1$.
   Many answers, such as $F = (-3x^2 - 1, 1)$

4. Consider the surface $S$ in $\mathbb{R}^3$ given parametrically by $x = u\cos(v)$, $y = u\sin(v)$, and $z = u$. Let $(u, v)$ range through the domain $D = \{(u, v)|0 \leq u \leq 1, 0 \leq v \leq 2\pi\}$.
   A. Graph $S$. Mark the grid curves $u = 1$ and $v = 0$.
      This is a cone. $u = 1$ is upper rim and $v = 0$ is a line running up the side.
   B. Find the surface area (for $(u, v) \in D$).
      $SA = \sqrt{2}\pi$
   C. Let $C$ be the grid curve $v = 0$, $0 \leq u \leq 1$. Find $\int_C 1ds$. What physical quantity does this integral represent?
      This is the arclength of the line segment from $(0, 0, 0)$ to $(1, 0, 1)$ and so has length $\sqrt{2}$.

5. Suppose $F = (yz, yz^2, z^3e^{xv})$. Suppose $S$ is the part of the sphere $x^2 + y^2 + z^2 = 5$ above $z = 1$ oriented upwards. Find $\int_S \text{curl} F \cdot dS$.
   Answer $= -4\pi$

6. Suppose $F = (3x, xy, 2xz)$. Suppose $S$ is the boundary of the cube $\{(x, y, z)|0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$. Find the flux of $F$ across $S$.
   $\text{Flux} = 9/2$. 