1. Let \( f(x) = \begin{cases} e^x + e^{-1/x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases} \) and assume that \( f^{(n)}(0) = 1 \) for \( n = 0, 1, 2, 3, \ldots \).

(a) What is the Maclaurin series for \( f \)?

(b) What is the interval of convergence of the Maclaurin series?

(c) For what values of \( x \) does \( f(x) \) equal the sum of the Maclaurin series?
(a) Find the Maclaurin series for the function \( f(x) = x \cos^2 x \). Write the result in closed form—as a sum \( \sum_{n=0}^{\infty} a_n x^n \). (Hint: \( \cos^2 x = \frac{1}{2}(1 + \cos(2x)) \).)

(b) Find the sum of the series \( \sum_{n=0}^{\infty} \left( \frac{x^2 + 1}{3} \right) ^n \) as a function of \( x \). What is the interval of convergence of the series.
3. (a) Find the first 5 terms of the Maclaurin series of the function \( f(x) = (1 - 3x^2)^{-1/3} \).

(b) If the first 6 terms of the Maclaurin series of the function \( f(x) = \frac{1}{\sqrt{1 - x^2}} \) are

\[
(1 - x^2)^{-1/2} = 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{5}{16}x^6 + \frac{35}{128}x^8 + \frac{63}{256}x^{10} + \ldots,
\]

find the first 7 terms of the Maclaurin series of the function \( f(x) = \arcsin(x) \).
4. The Maclaurin series for \( f(x) = \cos x \), \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \), converges for all \( x \), \( -\infty < x < \infty \). Show that the series converges to \( f(x) = \cos x \) for all \( x \).
(a) Find the Taylor series expansion for $f(x) = \ln(x)$ at $a = 4$. Write the result using summation notation.

(b) Find the first four terms of the Taylor series expansion for $f(x) = x^2 \ln(x)$ at $a = 4$. 
6. For what values of $x$ does the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}$ converge.

7. Find the Taylor series of $f(x) = x^3 - 2x + 4$ at $a = 2$. 