Pries M161: Rock the test! Review sheet for Parametric and Polar

1. Sketch the curve of the polar equation \( r^2 = 4 \sin(2\theta) \).

2. Find the area enclosed in one loop in one loop of the lemniscate \( r^2 = 4 \sin(2\theta) \).
3. Sketch the curve of the parametric equations \( x = t(t^2 - 3), \ y = 3(t^2 - 3) \) for \(-2 \leq t \leq 2\). Indicate with an arrow the direction in which the curve is traced as the parameter \( t \) increases.

4. Find the points on the curve where the tangent line is horizontal or vertical. Draw these tangents on your plot in problem 3.

5. Set up the integral to determine the length of the curve given in problem 3.
6. Find the cartesian form of the polar curve \( r \cos(\theta) + r \sin(\theta) = 1 \).

7. Find the polar equation for the Cartesian equation \( x^2 + (y-3)^2 = 9 \). Write your result in the form \( r = f(\theta) \), i.e. solve for \( r \).

**Formulae to understand**

Polar in terms of Cartesian: \( r = \sqrt{x^2 + y^2} \) and \( \theta = \tan^{-1}(y/x) \).

Cartesian in terms of polar: \( x = r \cos(\theta) \) and \( y = r \sin(\theta) \).

Slope for parametric: \( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \) and second derivative \( \frac{d^2y}{dx^2} = \frac{d(y'/dt)/dt}{dx/dt} \) where \( y' = \frac{dy}{dx} \).

Slope for polar \( \frac{dy}{dx} = \frac{(dr/d\theta)\sin(\theta)+r\cos(\theta)}{(dr/d\theta)\cos(\theta)-r\sin(\theta)} \).

Arclength for parametric: \( L = \int_{\alpha}^{\beta} \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt \).

Arclength for polar: \( L = \int_{\alpha}^{\beta} \sqrt{r^2 + (dr/d\theta)^2} \, d\theta \).

Area for polar: \( A = \int_{\alpha}^{\beta} (1/2) r^2 \, d\theta \).

Area inside \( r_2 \) and outside \( r_1 \): \( A = \int_{\alpha}^{\beta} (1/2)(r_2^2 - r_1^2) \, d\theta \).