Calculate the following integrals. You must show your work. These integrals must be integrated analytically. If you just give the result from your calculator, you will get zero credit.

(a) \( \int \frac{x^{1/3}\ln x}{x^{1/3}} \, dx \)

\[ \int f'g \, dx = fg - \int fg' \, dx \]

\[ = \frac{1}{1 + \frac{1}{3}} x^{1/3} \ln x - \int \frac{1}{3} x^{2/3} \cdot \frac{1}{x} \, dx = \frac{3}{2} x^{2/3} \ln x - \frac{3}{4} x^{4/3} + C. \]

(b) \( \int \frac{x}{x - 1} \, dx = \int \left(1 + \frac{1}{x - 1}\right) \, dx = x + \ln|x - 1| + C \]

reducing to proper term.

(c) \( \int \frac{1}{x^2 - 1} \, dx \)

partial fraction.

\[ \int \frac{1}{(x+1)(x+1)} \, dx = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1) + B(x-1)}{(x+1)(x-1)} \]

\[ 1 = 2A \quad A = \frac{1}{2} \]

\[ 1 = -2B \quad B = -\frac{1}{2} \]

\[ \int \left(\frac{1}{x-1} + \frac{-1}{x+1}\right) \, dx = \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C \]

\[ = \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + C. \]
2. Calculate the following integrals. You must show your work. These integrals must be integrated analytically. If you just give the result from your calculator, you will get zero credit.

(a) \( \int_1^2 x^4 \ln x \, dx \) 
   - Integration by parts.
   - Check basic examples.

(b) \( \int_3^2 \frac{1}{1 - x^4} \, dx \) 
   - Partial fraction
   - \( \frac{1}{(1-x)(1+x)} \)
   - Check.

(c) \( \int_0^2 \frac{1}{\sqrt{x}} \, dx \) 
   - Basic integration formulas

(d) \( \int \frac{1}{(1-x^2)^{3/2}} \, dx \) 
   - Basic form
   - The substitution 1
   - \( x = \sin t \)
   - \( dx = \cos t \, dt \)
   - \( \int \frac{1}{(1-x^2)^{3/2}} \, dx = \int \frac{\cos t}{\cos^3 t} \, dt = \int \frac{1}{\cos^2 t} \, dt \)
3. Calculate the following integrals. You must show your work. These integrals must be integrated analytically. For definite integrals give exact answers—no calculator approximations. If you just give the result from your calculator, you will get zero credit.

(a) \( \int^{3}_{2} \frac{1}{x^{1/3}} \ln x \, dx \)

(b) \( \int^{\sqrt{2}}_{0} \frac{1}{\sqrt{1-x^2}} \, dx \)

\[ \text{Trig substitution:} \]

\( \sin^{-1}(x) + C \)

(c) \( \int^{3}_{2} \frac{x}{x^2 - 1} \, dx \)

\[ \text{Substitute:} \]

(d) \( \int \sin^3 x \cos^2 x \, dx \)

\[ = \int \sin x \left( 1 - \sin^2 x \right) \cos x \, dx \]

\[ = \int (\sin x) \, \cos x \, dx \quad \text{substitute:} \]

\[ \frac{1}{2} \cos^2 x \quad \text{and} \]

\[ = \frac{1}{2} (x + \frac{1}{2} \sin(2x)) + C \]
4. Solve the following differential equation: \( \frac{dy}{dx} = \frac{1-x}{xy} \), \( x > 0 \), \( y(1) = -4 \). (Solve for \( y \)).

\[
y \cdot dy = \frac{1-x}{x} \, dx
\]

\[
\frac{y^2}{2} = \int \frac{1-x}{x} \, dx + C = \ln|x| + C.
\]

\[
y = \sqrt{2x + 2\ln|x| + C}
\]

\[
y(1) = -\sqrt{2 + C} = -4 \quad \Rightarrow \quad C = 14
\]

5. Find the area under one arch of the cycloid \( x = t - \sin t \), \( y = 1 - \cos t \), \( 0 \leq t \leq 2\pi \).

\[
(x, y) = (t - \sin t, 1 - \cos t)
\]

\[
dA = y \cdot dx = y \frac{dx}{dt} \, dt.
\]

\[
A = \int_0^{2\pi} (1 - \cos t)(1 - \cos t) \, dt = \int_0^{2\pi} (1 - \cos t)^3 \, dt
\]