14) a) Determine the number of (monic) irreducible polynomials of degree 6 over $\mathbb{F}_5$.
b) How many of these polynomials stay irreducible over $\mathbb{F}_{25}$?
c) Let $f \in \mathbb{F}_5[x]$ be irreducible of degree 6. What are the degrees of the factors $f$ over $\mathbb{F}_{625}$?

15) Let $f := x^5 + 3x + 2$ and $g := x^7 + 2x^4 + x + 1$ over the field $\mathbb{F}_7$. Calculate $f^{5000} \mod g$ without involving polynomials of degree $> 14$.
(You may use a computer, but not a `PowerMod` function.)

16) Perform a squarefree factorization on the polynomial
\[
\begin{align*}
&x^{38} - x^{37} - x^{35} - x^{34} - x^{33} + x^{32} - x^{31} - x^{30} - x^{29} - x^{28} - x^{27} + x^{24} - x^{21} + x^{19} \\
&+ x^{15} - x^{12} + x^{11} - x^8 + x^7 + x^5 - x^4 + x^3 - x^2 - x - 1 \in \mathbb{F}_3[x]
\end{align*}
\]

17) Perform a distinct-degree factorization on the (larger degree) squarefree factor of the polynomial from problem 16.

18) Factorize the polynomial from problem 16 into irreducible factors over $\mathbb{F}_3$, using the method of CANTOR and ZASSENHAUS.

Remarks on Problem 8:  

a) Let $\rho \in \text{Gal}(K/F)$. Then
\[
\rho(N(g)) = \prod_{\sigma \in \text{Gal}(K/F)} \rho(\sigma(g)) = \prod_{\sigma \in \text{Gal}(K/F)} (\rho \sigma)(g).
\]
If we set $\tau = \rho \sigma$, while $\sigma$ runs through all group elements, also $\tau$ runs through all group elements, the factors just get permuted. Thus $\rho(N(g)) = \prod_{\tau \in \text{Gal}(K/F)} \tau(g) = N(g)$ is invariant under the Galois group and thus $N(g) \in F[x]$.
b) We have to make the extra assumption that $K = F(\theta)$ – otherwise we have to amend the definition of the norm to exclude duplicates.
As $f(y) = \prod(x - \theta_i)$ one of the identities for the resultant gets us $\text{res}_y(f(y), g(x,y)) = \prod_i g(x, \theta_i)$. On the other hand, the images $\sigma(\theta)$ are just the $\theta_i$ which shows equality.
c) As $g$ is a factor of $N(g)$, every root of $g$ is root of $N(g)$. 